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Functional Competence in Mathematics—Its Meaning and Its Attainment*

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INTRODUCTION

A COLLEGE professor has said that if all the statisticians and economists of the nation could be laid out in a straight line, end to end, they would not reach a conclusion. Experienced classroom teachers will readily admit that this disconcerting opinion applies even more strongly to our educational policymakers. Their chronic inability to agree on a definite program for any length of time, or else the haste with which they often disavow the very plans they acclaimed so enthusiastically only yesterday, has reduced the edutional scene to a condition bordering on chaos. Externally, to be sure, we have much to be thankful for. Magnificent new school buildings have been constructed. The public support of education is becoming more dependable. Enrollment figures have risen to spectacular heights. Our children and young people usually like to go to school. On the whole, they are satisfactorily nourished and their health is carefully guarded. These are all great accomplishments.

Internally, however, the situation is quite different. Four years ago Professor Dewey, for two generations the fountainhead of our dominant educational philosophy, in a widely discussed magazine article offered this shattering comment:

We agree that we are uncertain as to where we are going and where we want to go, and why we are doing what we do.[1]

Few persons acquainted with actual conditions will fail to endorse Professor Dewey's caustic analysis. Is it not a fact that we have lost faith in the miracle formulas that have been poured out upon us in such profusion? Or have we more than nebulous notions as to what is the real function of our secondary schools and of our colleges? An endless stream of curricula, now estimated at a total of nearly 100,000, has been thrown at the teachers, only too often to be discarded after a brief, ineffective trial. Curriculum-building has been one of the chief indoor sports of education. Countless objectives, criteria and approaches have been evolved. But we are still rehearsing the relative merits and claims of vocational training and of a liberal education. And we are still retaining, in spite of numerous efforts at reform, a colonial, disintegrated type of school organization which definitely prevents

* Based on an address delivered at the Indianapolis meeting of the National Council of Teachers of Mathematics, April 3, 1948.

[1] From "Challenge to Liberal Thought," by John Dewey, in *Fortune*, August, 1944, p. 155. See, also, Dewey, John, *The Way out of Educational Confusion*, Inglis Lecture, p. 40, Harvard University Press, Cambridge, 1931.

really sequential and functional curricula. On the administrative side, we have abandoned the idea of enforcing even the most modest standards. The escape mechanism of automatic promotion has been sanctioned on a wide front. With the aid of a mechanistic measuring rod, the "unit," all subjects have been given equal rank. They have become the interchangeable parts of the educational machine. It is generally admitted that under these circumstances both high school and college diplomas have become mere attendance certificates without definite educational connotations. Last, not least, many thousands of teachers are entering our ranks without an adequate professional equipment.

It is in this setting that school mathematics has been trying to maintain its precarious existence. Along with other "old-line subjects," it has been out of step with the controlling educational slogans and policies. It cannot be derived from "immediate experience," nor can it be "adapted" indefinitely to "individual needs and interests." It demands honest, cumulative achievement and a continuous regard for progressive mastery. Hence it has become a barely tolerated school enterprise. It seems that our recent war experience has taught us nothing regarding the indispensable role of mathematics in human affairs. The prevailing doctrine is that school mathematics should be limited to the unavoidable rudiments dictated by social utility. As a permanent ingredient in any program involving genuine literacy and cultural orientation, it is being ignored or is being marked for extinction. Arithmetic has been "moved up" and is already becoming a college subject. "Academic" mathematics has been made an elective "for the few." Various substitutes are competing to fill the vacuum created by its elimination as a required subject. How the colleges are facing this crisis, is a sight wonderful to behold.

Now, is all this the inevitable outcome of mass education, so ardently heralded as the great panacea of the modern age? If not, how shall we react to the present situation? In particular, "whither mathematics?"

I. The Long Struggle for Functional Curricula

First of all, it seems necessary to examine, once again, the climate of opinion that surrounds mathematics today. In other words, what is the picture that prominent educators are creating as to the significance of mathematics? For whether this picture is true or false, it certainly determines the actual status of mathematics in our schools and it underlies the issue of functional competence.

To be sure, this whole story has been rehearsed a great many times in our professional literature. For at least two decades, certain critical appraisals and demands on the part of influential educators have recurred with monotonous regularity.[2] They have followed a definite pattern. The standard propositions which their authors keep repeating may be summarized as follows:

1. The mathematical curriculum, "for all but the few," should stress only those minimum essentials which are actually needed in everyday life.

2. All the mathematics taught in the school should be derived from actual life situations.

3. Academic mathematics of the usual type does not "meet the needs" of the vast majority of our young people, "the other 85%." It is largely "non-functional," and should be reserved "for the few."

In recent years a fourth proposition has been added by a very vocal group:

4. "All but a few" of our pupils can get

[2] For a discussion of such critical comments, see, for example, 1) Reeve, W. D., "Attacks on Mathematics and How to Meet Them," in the *Eleventh Yearbook of the National Council of Teachers of Mathematics;* 2) Betz, William, "The Necessary Redirection of Mathematics," in The Mathematics Teacher, April, 1942, p. 149; 3) Reeve, W. D., "Mathematics in the Post-War Period," in *Scripta Mathematica*, XI, 1945, pp. 275–307.

along without any mathematics in the secondary schools.

Before we submit some characteristic quotations of recent date, by way of illustrating this position, the following corrective observations seem warranted.

First, the reductionism advocated for mathematics is also operating in every other school subject. Thus, we now have English without grammar and literature, social studies minus history, science without measurement, and so on. But functional competence simply does not grow in such an impoverished soil.

Second, the frantic determination to protect "all but the few" against learning anything beyond the minimum essentials, in this most perilous era, gravely endangers a democracy like ours. It reverses the policies of the founding fathers and nullifies their high hopes. Wisdom, not ignorance, is the surest foundation of both freedom and prosperity. "Where there is no vision, the people perish."

Third, no other country has seen fit to recede from a *maximum* emphasis on mathematics. In little Switzerland, for example, every boy in high school is expected to complete a respectable unit of work in the calculus. Is that a sign of educational backwardness?

Fourth, more than three decades ago, according to Professor D. E. Smith, our high schools were at least two years behind the mathematical goals reached in other leading countries. Today the gap is much greater.

Fifth, are we prepared to assert that all other nations are mistaken in their high regard for mathematics, or that American boys and girls alone are totally unable to do what is readily accomplished elsewhere?

Sixth, have we forgotten that, as Professor Hotelling has pointed out, "there is no surer key to unlock all sorts of doors than mathematics?" What right have we to close these doors to "all but the few?"

Seventh, for nearly six decades we have had unceasing efforts at reform in mathematics. Among the milestones in this epic struggle we may mention the work of the Committee of Ten, in 1892, the publications of the International Commission, beginning in 1908, the Report of the National Committee in 1923, the twenty Yearbooks of the National Council of Teachers of Mathematics, the Joint Commission Report of 1940, and the reports of the Commission on Post-War Plans. Is it not true that in the past these efforts have almost regularly been ignored or even frustrated by our educational leaders? And yet those men keep on claiming that we have done little or nothing to improve the mathematical situation. Where have they been all these years? If we had had only an ounce of a more genuine type of cooperation from the educators and administrators, we should have had truly functional mathematical curricula long ago.

Eighth, why do our normal schools and colleges fail so completely in supplying the kind of training and leadership which is needed so badly in combatting the chaos described above? Without their assistance we cannot hope to develop and maintain functional curricula.

It remains to look at one or two critical pronouncements of the sort mentioned above. Thus, a California educator writes,

> Anyone who looks at the mathematics program as offered by the typical high school finds a very close resemblance to the offerings of a century ago. It is a sober fact that we have made but minor changes in the total program of mathematics, whereas our school population has undergone a wholesale revolution, not to mention the social, economic, and technological changes in the world about us.
>
> Without question the mathematics "sequence" serves a very worthwhile purpose for a selected group of students who will continue their education particularly in mathematics, science, and engineering. However, the other 85% have an altogether different set of needs and interests in mathematics. The schools specifically, and society generally can be indicted with the charge that very little has been done to make mathematics palatable, interesting or worthwhile to the majority of our secondary school population.
>
> It is becoming more evident that a solution to this problem lies in a totally new program of

mathematics for the non-college student, and perhaps even for a portion of the college bound students. Instead of drawing from the organized body of the science of mathematics, the new program should result from an analysis of the life situations and experiences that students will likely have during and after their school training. The results of such an analysis must then be organized into courses (required by mass education) and fitted into the core and elective portions of the curriculum.[3]

A Midwestern Superintendent of Schools, after attending a conference of suprintendents in Chicago, made the following statement to his teachers of mathematics:

Since only 12% of the pupils who study mathematics are ever going to college, we do not need to worry any longer about teaching mathematics in the secondary school.[4]

Even more surprising, however, is a set of declarations emanating from so formidable a body as the Commission on Life Adjustment Education for Youth, recently organized by the United States Office of Education. Here we have the prevailing climate of opinion in almost crystalline pureness. From comments published in the periodical literature we quote the following:

Only four out of ten U. S. children finish high school, only one out of five who finish high school goes to college. But most of the 25,000 U. S. high schools were still acting as if all their kids intended to go to college. Studebaker believes that educational reverence for the "white-collar myth" produces frustrated and maladjusted citizens. Why not frankly admit that most girls would be housekeepers and most men mechanics, farmers and tradespeople—and train them accordingly?

The commissioner invited nine prominent educators to Washington to help figure out a plan. Last week his group, the Commission on Life Adjustment Education for Youth, made its first suggestions.

Said the Commission:

Every "life-adjusted" youth needs to master practical English, social science, physical education, basic science.

It is a waste of time for most high school students to read *Il Penseroso, Ivanhoe, Silas Marner* and other compulsory classics. It would be enough for many to secure "sufficient competence in reading to comprehend newspapers and magazines reasonably well." Only a gifted few can achieve any real understanding of algebra or geometry. It should, therefore, be a matter of choice whether a student takes algebra, literature, Latin, foreign languages.

For these courses, students should be allowed to substitute part-time jobs under supervision—in department stores, drug-stores, etc. (Says Studebaker: "The youth adjusted to life is adjusted to his job. . . . ") High schools should add courses in homemaking and job-hunting.

High schools in 35 states are already working out this kind of "life adjustment" education. The next step, said the Commission, is to get everybody doing it.[5]

What shall we say to all this? In the light of the Guidance Report of the Commission on Post-War Plans, which enjoyed the hearty endorsement of the Office of Education, is it not extremely queer that basic mathematics is completely ignored? Apparently, our millions of "housekeepers," and our army of "mechanics, farmers, and tradespeople" are expected to be "life-adjusted" without a scrap of mathematical training. Moreover, has it ever been divulged how the high school will be able to segregate its entering pupils on the basis of "the few" and "the many"?

By a strange irony of fate, the doctrine of "the few" was boldly exploded, almost on the same day, by the President's Commission on Higher Education. This Commission, by way of contrast, advocates a vast *increase* in college enrollments. The colleges are to plan for a minimum enrollment, by 1960, of 4,600,000. However, further "adjustments" of all curricula are urged, presumably in the direction we have pointed out. The reaction of the colleges to the revolutionary plan of admitting everybody, irrespective of qualifications, is one of cautious waiting. Sardonic comments are beginning to appear. As one college thinker intimates, this plan of "educational inflation" will increase the number of sheepskins, but will not reduce the number of sheep.

[3] Siemens, Cornelius H., "Basic Mathematics in the Secondary Schools," in *The California Mathematics Council Bulletin*, November, 1946, p. 7.

[4] Reported by Reeve, W. D., Eleventh Yearbook, *op. cit.* p. 7.

[5] From *Time* magazine, December 15, 1947, p. 64.

II. The Meaning of Functional Competence in Mathematics

In current educational discussions the term "functional" is used in the sense of "practical," "useful," "based on experience," "applicable," "socially valuable," and the like. In one way or another, this interpretation is based on our dominant philosophy of education, that of pragmatic instrumentalism and of immediate experience. We shall maintain that this interpretation is incomplete and misleading, and that a decided reorientation is necessary.

Mathematics is the oldest of all the sciences. It dates back to the dawn of human history. As a field of interest and study its record covers a period of four thousand years. Surely, some valid lessons should have emerged by this time as to the role of mathematics in human affairs and as to the best ways and means of approaching it. That is certainly the case. In fact, it is these lessons that underlie the principal thesis of this paper, which is to the effect that there are *three* main components of functional competence in mathematics, not one of which can be neglected without fatal consequences.

In developing this thesis, we shall make use of a simple yet helpful allegory. We may look upon mathematics as a majestic old tree whose roots, trunk, and branches will serve to suggest the message we need for our purposes. The roots of the tree represent the human needs out of which mathematics grew. The trunk of the tree denotes the imposing framework of mathematics, while the branches represent the vast domain of mathematical applications. Since the story of the beginnings of mathematics is a rather familiar one, we shall turn at once to a consideration of the trunk of our mathematical tree.

The Framework of Mathematics—The Trunk of the Tree. Briefly, the framework of mathematics rests on its basic concepts, principles, and facts, on its tested rules of computation, its established methods of solving problems, and its modes of thinking. It took many centuries to create this imposing structure. By slow degrees it took the form of a system which includes the fields now known as arithmetic, geometry, algebra, and trigonometry. In modern times the enormous expansion of mathematical research, of industry and science, of technology and commerce, caused the mathematical tree to grow more than in all the ages of its earlier history.

Now, this dramatic development was not effected by a few bright individuals. On the contrary, mathematics is the product of *racial experience*. It is the joint contribution of the ages, the collective masterpiece of a large number of thinkers. Hence it is quite certain that this system, even in its most elementary ranges, cannot be rediscovered or rebuilt by the individual pupil or school solely on the basis of personal experience or of "individual needs and interests." That would require many years of a most extraordinary kind of life.

No one has stated this conclusion more forcibly, in recent years, than Professor Harry G. Wheat of West Virginia University. In the following significant quotation, due to his pen, the word "arithmetic" has been replaced by the broader term "mathematics." this passage then reads as follows:

> Mathematics is a system of ideas. It is not a collection of objects. It is not a set of signs. It is not a series of physical activities. Mathematics is a system of ideas. Being ideas, mathematics exists and grows only in the mind. It does not flourish in the world of things. It does not arise out of sensory impressions. It has nothing to do with the amount of chalk dust forty pupils can raise in a schoolroom in thirty minutes. Mathematics exists and grows only in the mind. Being a system, mathematics must be taught as a system. It is not an outgrowth of the individual's everyday experiences. It is not learned according as the interests or the whims of pupils may suggest. It is not anyone's personal discovery or invention. Mathematics must be taught as a system.

Hence we may state, with complete confidence, that **the first component of functional competence in mathematics is the systematic study, within a desired range, of the underlying mathematical**

THE TREE OF MATHEMATICS

This drawing was made for the purposes of the present article, under the direction of the author, at the Cleveland School of Art.

It will be noted that its underlying idea resembles that of the famous poster entitled "The Tree of Knowledge." (see page 247)

concepts, principles, skills, and modes of thinking.

The Applications of Mathematics—the Branches of the Tree. Let it be recalled that mathematics grew out of life situations, namely, the human needs that first led primitive people to count and to measure. That is absolutely undeniable. Moreover, it was the problem situations arising in such everyday activities as farming and building which caused the mathematical tree to develop its flourishing branches. It is true that the "system" of mathematics was gradually perfected by ancient and modern thinkers, by philosophers, scientists, astronomers and professional mathematicians. But without the constant interaction of the concrete and the abstract, of theory and practice, the mathematical tree would have lost its sustaining contacts with the world of reality. Its branches would have withered and the tree would have ceased to grow.

The school cannot afford to ignore the vital lesson of this allegory. When it does, the mathematical structure becomes a domain of "inert ideas." None other than the late Professor Alfred N. Whitehead has issued the following classic warning against the danger of separating school subjects from their parent source:

> There is only one subject-matter for education, and that is Life in all its manifestations. Instead of this single unity, we offer children—Algebra, from which nothing follows; Geometry, from which nothing follows; Scince, from which nothing follows; History, from which nothing follows; a couple of Languages, never mastered. . . . Can such a list be said to represent Life, as it is known in the midst of the living of it?[7]

And so, we may safely assert that **the second component of functional competence in mathematics is a proper emphasis on the significant interrelations between mathematical theory and its many-sided applications.**

Hence the educators are on absolutely safe ground when they urge the importance of *social utility*. The decline of mathematics was caused very largely, though not entirely, by the "inert," abstract, seemingly useless character of our curricula, our textbooks, and our examinations. We have to face years of hard labor if we want to correct the errors of the past.

However, the educators have fallen into an even more serious blunder. They are actually proceeding on the naive assumption that we can have the branches of the tree without the tree itself, that we can competently *apply* mathematics with little or no regard for its basic ideas and techniques.

This error, too, must be uprooted, before further progress is possible. All attempts—and they have been numerous—to build a mathematical program entirely on the basis of "life situations," or "practical problems," or "areas of experience," or "social backgrounds," or "personal needs and interests," have regularly failed completely. A record of these failures is readily accessible.[8] The reason for these failures can be recorded in a single sen-

[7] Whitehead, Alfred N., *The Aims of Education and Other Essays*, the Macmillan Company, 1929, p. 10. This passage was chosen as a central motto by the Educational Policies Commission for its widely publicized volume on *The Purposes of Education in American Democracy* (1938).

[8] Thus, the Perry movement in England, at its height nearly fifty years ago and widely advertised in this country, never succeeded. The "applied-problem movement" of more than thirty years ago, though far more conservative, petered out because technical backgrounds cannot be improvised in the classroom. The *Social Arithmetic* of McMurry and Benson (1926), the most noteworthy attempt to "socialize" arithmetic, was too forbidding and too unpedagogic to make a permanent impression. Similar remarks apply to more recent adventures of this sort. It seems, however, that we are not profiting from this persistent record of failures. Extremists who are still devoted to an exclusive platform of socialization, "personal experience," and the like, should be urged to study pertinent discussions of this subject, such as the following:

Rugg, Harold, *American Life and the School Curriculum*, Ginn and Company, 1936, p. 203; Dewey, John, *Experience and Education*, The MacMillan Company, 1938, p. 111; also, "Democracy and the Curriculum," the *Third Yearbook of the John Dewey Society*, D. Appleton-Century Company, 1939, pp. 414 ff.

tence: "*Mathematics is a system of ideas, and must be taught as a system," but no one has ever succeeded in organizing the more or less accidental "experiences" or alleged needs of thirty or more pupils into a system co-extensive with that of mathematics.*

The Mathematical Tree and the School. It should be obvious that the framework of mathematics and its applications constitute an *objective* domain which is based on racial experience and age-long endeavor. This domain exists quite independently of the learner's likes, dislikes, and abilities. He cannot create it. He can only try to understand it, and to use it effectively, within the limits of his educational goals.

But this learning process is another all-important part of our story. Today we know that it makes all the difference in the world how it is carried on. Many valuable chapters have been added to the psychology of learning. We have ample proof that "social utility" alone is not a substitute for a clear understanding and mastery of the basic concepts, skills, and methods of mathematics. It remains true, as Professor Dewey pointed out long ago, that

> Practical skill, modes of effective technique, can be intelligently, non-mechanically used, only when intelligence has played a part in their acquisition.[9]

We know, moreover, that "transfer" is not automatic, that mastery is not achieved in a day, and that motivation and interest are of tremendous importance.

Is it not clear, then, that we must add still another component to our growing definition of functional competence in mathematics? This *third* component, the *subjective* or *psychological* factor, may be formulated as follows:

Functional competence in mathematics is largely the outgrowth of a continuous and painstaking emphasis on the categories of understanding, mastery, and transfer.[10]

[9] Dewey, John, *How We Think*, D. C. Heath and Co., 1910, p. 52.

III. The Attainment of Functional Competence in Mathematics

That we are far from achieving functional competence in mathematics today, is attested by the nationwide evidence of a breakdown of mathematics in our schools and colleges. This deplorable situation has been brought about gradually by the inept attitudes and prescriptions of various pressure groups, including educational theorists, administrators, curriculum specialists, and other policymakers. It is the old story. When too many doctors work on a case, the patient often dies. A herculean effort will be necessary to bring back a semblance of health to the ailing subject of mathematics.

How, then, may we attain functional competence in mathematics?

Even a moderately detailed discussion of that question would require a lengthy volume. Instead, we must limit ourselves to the enumeration of the following "first-aid" measures that would probably be endorsed by the majority of classroom teachers who are familiar with the facts.

First, we must give strict and unremitting attention to the *three* components of functional competence that we have pointed out.

Second, we must counteract erroneous educational theories, such as the misleading doctrines of "immediate experience" and of "individual needs and interests."

Third, we must build *continuous* curricula.

Fourth, we must absolutely reject the policy of automatic promotion.

Fifth, we must develop a "two-track program" at the secondary level.

Sixth, we must eliminate or completely readjust such curricula as are obviously

[10] For a more detailed discussion of the ideas presented in the preceding pages, see Betz, William, "Functional Competence in Mathematics," and "Elements of a Functional Program in Mathematics," in *The Science Counselor* March, 1947, and June, 1947. The first of these papers was reprinted in the *Bulletin of the Kansas Association of Teachers of Mathematics*, October, 1947, and December, 1947.

faulty or based on wrong theories.

Seventh, we must have a radical correction of the unendurable arithmetic situation. In particular, we must get rid of the totally false postponement idea.

Eighth, we must insist on a far more adequate teacher-training program.

Ninth, we must stop our slavish subservience to the pronouncements of self-constituted experts having little or no classroom experience, in favor of carefully tested plans developed in genuine laboratory schools operated by real teachers.

Tenth, we must have a publicity campaign, sponsored by the National Council of Teachers of Mathematics and other affiliated groups, which shall acquaint the general public with the actual mathematical situation and its causes.

That looks like a long journey. It will be. It usually takes considerable time to cure a chronic illness. However, as was pointed out above, the reform movement in mathematics has already gone on for nearly six decades. As a result, the chief building stones for the construction of truly functional curricula have long been available. The yearbooks of the National Council, as well as many scores of reports or articles that have appeared in the pages of such publications as THE MATHEMATICS TEACHER and *School Science and Mathematics,* are full of vitally necessary information concerning almost every phase of our professional problems. Many individuals and committees have unselfishly furnished this valuable information. Why not, at long last, give real attention to it? Why act as though nothing whatever has ever been done?

Our main task, for quite a while, will have to be the eradication of the grievous blunders and obstacles which have been impeding our progress. Let us examine a few of them, not in a spirit of mere fault-finding, but of a genuine desire to dispel the fog that hangs over the mathematical scene.

In the first place, let us admit that as *teachers of mathematics* we have sinned for many years by giving almost exclusive attention to the *first* component of functional competence, to theory without application. In the eyes of the learner, we have been purveyors of "inert ideas." And the methodology prevailing in countless classrooms can still be described in two sentences: "For tomorrow take the next fifteen examples. The following twelve pupils may go to the board." As a result, we have had "muscular" mathematics. But, in the highly personal style of Professor Kilpatrick, "functional competence does not so come."

On the other hand, the advocates of "*socialized mathematics*" have been even more at fault by stressing only the *second* component of functional competence. Once again, "functional competence does not so come." Programs consisting of an array of "social problems" regularly exhibit serious weaknesses. The check list for functional competence published by the Commission on Post-War Plans shows how defective most of these plans are.

Very similar remarks apply to many of the *current one-year programs in "General Mathematics."* Detailed studies have revealed that quite commonly these plans represent a very arbitrary assortment of odds and ends.[11] They begin nowhere and

[11] See, for example, the master's thesis of Faith F. Novinger (unpublished, George Washington University). It offers a careful analysis of twenty-three recent textbooks designed for ninth-grade pupils, largely of the one-year general mathematics type. A brief abstract was published in THE MATHEMATICS TEACHER, April, 1942. The study revealed a state of confusion bordering on chaos. Thus, the amount of space given to arithmetic in these texts varies from 4 to 209 pages; in algebra it varies from 4 to 207 pages. Seven of the texts omit algebra altogether. In geometry the modal number of pages is 100. A similar study, made by a committee of Rochester teachers, involved an analysis of fourteen one-year terminal textbooks in ninth-year mathematics, all published since 1935. In arithmetic, the percentage of text pages varied from 0% to 94%; in algebra, the range extended from .5% to 72%; in geometry, from .5% to 37%; in trigonometry, from 0% to 15%; in correlated mathematics (mainly socialized applications), from 0% to 51%.

If anyone still doubts the necessity of a *two-*

they end nowhere. It is foolish to expect from them a genuine understanding and mastery of elementary mathematics.

Hence it should not surprise us that so competent a critic as Colonel W. E. Sewell of the War Department, in a recent address, found it necessary to say:

> Mathematics has gradually been removed from the various curricula until there is very little left that is useful or even recognizable. Many of the courses which are called mathematics are a disgrace to the name. They are designed for amusement, and anything which might be thought-provoking is carefully avoided.[12]

If we really desire to have a functional *two-track program* at the secondary level, our curriculum-makers and administrators must be prepared to set aside *at least two years* for a course in general mathematics intended for "the other 85%." Anything less than that will simply continue or repeat the futile contortions of the past.

This consideration of our major impediments would not be complete without a word about the appalling *arithmetic situation.* As the Commission on Post-War Plans stated so emphatically in its Second Report, the elementary school can and should furnish a really dependable foundation in arithmetic. *That such a foundation can be achieved in the first six grades, it would be utterly foolish to deny.* And without that kind of substructure the entire mathematical edifice collapses. That is what is happening today. The underlying causes of the present situation are not a secret. Aside from the prevailing educational philosophy and a lack of administrative support, we must attribute it in large measure to the arithmetic "experts." They have been guilty of unpardonable blunders, in spite of unending "research." Thus, they fell for Thorndike's *bond theory*, not heeding the insistent warnings coming from men like Wheeler, Judd, and Brownell. With equal docility and lack of critical judgment, they glorified "childlike activities and projects," "experience curricula," "socialization," and the like, when these dubious trends were vigorously denounced even by "progressive" educators.[13] To this day, after their rather sudden conversion to the "meaning theory," dating back to Dr. Brownell's article in *The Tenth Yearbook of the National Council of Teachers of Mathematics*, leading members of the arithmetic fraternity do not seem to understand it. Pictures are not a substitute for concepts and principles. Above all, these "experts" did not hesitate to give their endorsement to the most colossal aberration of the past sixty years, that of *postponing* arithmetic, all on the basis of alleged findings that were promptly discredited by the National Council and by competent students of

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year program "for the other 85%," let him study the "check list for functional competence" in the reports of the Commission on Post-War Plans, or Dr. Breslich's comprehensive list in The Mathematics Teacher, February, 1948, pp. 63 ff. Let him put after each topic the minimum number of class periods which it would certainly require, *in the light of actual experience.* The resulting totals will be found decidedly revealing.

A more recent, corroborating analysis of more than 50 textbooks in general mathematics for college purposes, supplemented by the responses to a questionnaire from more than 450 teachers of general mathematics, is that reported by Kenneth E. Brown in The Mathematics Teacher, November, 1946.

[12] See, "Mathematics in the Army Education Program," by Colonel W. E. Sewell, Chief, Education Branch, Information and Education Division, War Department, in *The American Mathematical Monthly*, April, 1947, pp. 195–200.

[13] The utter futility of an exclusive dependence on "activity" or "experience" programs in arithmetic is strikingly suggested by the classic monograph on "Opportunities for the Use of Arithmetic in an Activity Program," by Paul R. Hanna and Others, in *The Tenth Yearbook of the National Council of Teachers of Mathematics*, pp. 85–120. Enthusiasts who seem determined to repeat the errors of the past should be reminded of recurrent, incisive warnings by prominent educators. See, for example, Dewey, John, *Democracy and Education*, Chaps. XI–XII, The Macmillan Company, 1917; Rugg, Harold, and Shumaker, Ann, *The Child-Centered School*, pp. 129 and 131, World Book Co., 1928; Cobb, Stanwood, *New Horizons for the Child*, Chap. VIII, The Avalon Press, 1934; Bobbitt, Franklin, *The Curriculum of Modern Education*, McGraw-Hill Co., 1941, pp. 94 ff.

Activities and experiences, when introduced at the proper time, serve a very useful purpose by way of motivation and application. They are *not* a substitute for systematic instruction.

arithmetic.[14] *The support accorded to this totally unwarranted and terribly costly policy has virtually sabotaged six decades of mathematical reform.* Until this blunder is corrected, mathematics will continue to flounder and we shall be unable to attain truly functional curricula in mathematics.

Finally, it should not be overlooked that to this day almost the sole equipment of the standard mathematical classroom consists of the adopted textbook and a box of chalk. That being the case, the textbook is an all-important factor of our total problem. Can it be denied that *the textbook situation*, likewise, is far from ideal? To be sure, textbooks are written to be sold. Hence textbook writers, quite naturally, pay close attention to the current popular trends. But far too often our authors are mere followers, not leaders. That is why our textbooks so often reflect all the weaknesses that are troubling us. It is not necessary to catalogue these flaws. They are too obvious. It must be admitted that the new textbooks are beautifully printed, and they have lovely pictures. Increasingly, however, our mathematical texts have favored one-page snapshots, loosely strung together without apparent continuity or coherence. Once more, "functional competence does not so come." Criticisms such as the following from the pen of a leading college professor, can hardly be ignored:

> In my opinion our standard textbooks train the students in a limited number of routine processes and rarely call upon them to carry out original logical thought processes. Indeed some of the most widely accepted texts are little more than cookbooks in which the student learns how to find the answer in the book by following Step One, Step Two, and Step Three.[15]

Conclusion

We have looked at a picture which is no doubt perfectly familiar to every experienced teacher of mathematics. It is that of a battle between two sharply contrasting positions regarding the educational role of mathematics. We feel that we have a right to expect a reorientation on the part of our educational policymakers. There are indications that such a change of mind is actually impending. It is a distinct pleasure to be able to quote such corroborating statements as the following, recently made by a California educator, Dr. A. John Bartky, the new Dean of the School of Education at Stanford University. He wrote,

> Mathematics is a definite part of living and culture. . . . We have no right to deny any student the opportunity to see this side of human culture, nor dare we cut him loose from school without the necessary mathematical tools. . . . Our progress in all the sciences has to a large extent been dependent upon the use of the tool called "mathematics."
>
> You counselors who advise students to discontinue their studies of mathematics, have you ever stopped to think how much you are limiting their future? The student who drops high school mathematics at the same time drops medicine, engineering, chemistry, biology, physics, economics, social science and psychology. . . . The problem of guidance in mathematics is not just a matter of using a stop and go sign on the pupil. It is a highly complex process which must concern itself with the teacher as well as the pupil.

[14] Those who are concerned with the teaching of arithmetic in the elementary school cannot be urged too strongly to become thoroughly familiar with the authoritative literature pertaining to the postponement controversy. Among the titles of central importance are the following:

1. Brownell, William A., "Arithmetic in Grades I and II," Duke University Press, 1941. (This is the cardinal research monograph relating to the basic factors of the controversy. It is fully documented. The final bibliography contains 60 titles.)
2. Brownell, William A., "A Critique of the Committee of Seven's Investigation on the Grade Placement of Arithmetic Topics," *Elementary School Journal*, March, 1938, pp. 495–508.
3. Washburne, Carleton W., "The Work of the Committee of Seven on Grade-Placement in Arithmetic," *Thirty-Eighth Yearbook of the National Society for the Study of Education*, 1939, pp. 299–324.
4. Dickey, John W., "Readiness in Arithmetic," *Elementary School Journal*, April, 1940, pp. 592–598.

Significant articles on this subject have also appeared from time to time in The Mathematics Teacher, such as those of Sueltz (October, 1937), Thiele (February, 1938), and Buswell (May, 1938).

[15] Allendoerffer, C. B., "Mathematics for Liberal Arts Students," in *The American Mathematical Monthly*, December, 1947, p. 574. Dr. Allendoerffer, of Haverford College, is the Second Vice-President of the Mathematical Association of America.

The teacher must be taught that she has a responsibility to teach mathematics to all her students. She must feel the same way about mathematics as the elementary school teachers feel about reading. It has to be taught! . . . No secondary mathematics teacher should deign to take the responsibility of recommending "no more mathematics" for any student.

In guiding students in secondary mathematics the assumption must always be "I shall assure myself that this student has every opportunity to learn as much mathematics as possible".[16]

In any case, the battle must go until the victory is won and mathematics is at last accorded the place in the educational scheme which is in harmony with its cultural importance and its overwhelming significance in the modern world.

[16] Bartky, A. John, "Guidance in Mathematics," in *The California Mathematics Council Bulletin*, November, 1946, pp. 3 ff.

WORKSHOP FOR TEACHERS IN MATHEMATICS, SCIENCE, AVIATION

The Ohio State University, Columbus 10, Ohio

June 22-July 28, 1948

A workshop for teachers and supervisors concerned with improving the programs of mathematics, science or aviation in their own school systems is being offered by Ohio State University this summer. It has been planned particularly for teachers and administrators in junior and senior high schools, and is part of the summer session offering of the Department of Education. The workshop will be given during the first six weeks term, June 22 to July 29. Teachers with appropriate backgrounds may register in the workshop for approximately half of a full term load for graduate credit, and take other graduate or undergraduate courses for the remainder.

There will be two major types of experiences provided; individual or small-group projects, and common activities involving the group as a whole. Problems of concern to each participant will be studied in connection with the concept of a continuous program of mathematics and science from the first through the twelfth grade. Students will work on such problems as planning courses of study, working out guide sheets and tests, carrying out laboratory and field work, improvising apparatus, locating reference materials, and trying out demonstrations and visual aids. Provision has been made for first hand experiences with demonstrations, laboratory equipment, and field work in each of the areas. Common activities of the whole group include such activities as lectures, campus visitations, motion pictures, exhibits, discussion groups, off-campus trips, operation of School Link Trainer, and airplane flights.

In addition to the regular members of the workshop staff which includes Professors H. P. Fawcett, J. S. Richardson, and G. P. Cahoon, a number of visiting experts have been invited to participate. These include Dr. Philip Johnson, Specialist for Science, U. S. Office of Education, Washington, D. C.; Wm. Konicek, Link Aviation, Binghamton, New York; Horace Gilbert, Civil Aeronautics Administration, Assistant to Regional Administrator, Chicago; Roy Mertes, Director, School and College Service, United Air Lines, Chicago.

For a summer school bulletin, write to Registrar, The Ohio State University, Columbus, Ohio. For further information relating to this workshop, write G. P. Cahoon, Arps Hall, The Ohio State University, Columbus 10, Ohio.

Mathematics in Action

By Florence Barber
Education Section, U. S. Savings Bonds Division, Treasury Department

Teaching mathematics as a content subject with social aims, instead of solely as a tool subject which it used to be, has pointed up the need for classroom activities that provide effective experiences and learning situations. Among the best of the functional teaching aids now available to mathematics teachers is the U. S. Treasury School Savings Program.

With weekly Stamp Day as a base of operations, supplemented by School Savings teaching aids, this nationwide program in the hands of a skillful teacher can help pupils make many economic applications of mathematics—and at the same time learn thrift and good money management.

The School Savings Program provides three essentials to current requirements in the teaching of mathematics: meaningful activities, projects and problems arising out of the pupil's own experience, and the certainty that each pupil may gain a sense of achievement through his personal savings or through the leadership he gives to a project which results in the improved welfare of his group—or both.

Effective supplements to Stamp Day activities are the teaching aids recently published by the Education Section of the Treasury:[1]

Both will help teachers develop in students the skills and understandings which today's concept of adult citizenship demands.

The principal of a big-city elementary school recently wrote that his school was using *Lessons in Arithmetic Through School Savings* in the following ways:

1. For oral and written arithmetic
2. To teach money values
3. For oral and written reading lessons
4. For oral English drill
5. To teach new words in spelling
6. For drawing and coloring lessons
7. To teach letter form
8. As a means of instilling thrift, patriotism, ambition, etc.

"It has been very helpful and the children have enjoyed working with it," he added. "It has stimulated interest in School Savings generally."

Through all the grades and in high school, weekly Stamp Day, especially when its major responsibilities are carried by students, provides the kind of repetitive practice which is rich in meaning and guards against forgetting.

Buying and selling Stamps, making change, balancing accounts, keeping records help students form the habit of *using* arithmetic. Arithmetical skills and ideas, once their usefulness is demonstrated to the individual student's satisfaction, become the habitual way of meeting and dealing effectively with many life situations.

That the School Savings Program can speed up the learning process is amply demonstrated by the experience of a 2nd-grade teacher in Morrisville, Pa., whose 26 pupils learned to read money expressions in the hundreds of dollars between October when their Savings Program began and the end of February when their combined savings reached $334.80. After this teacher had written the first week's Stamp sales total on the board, the children insisted on seeing the cumulative total on the board each week. In addition to the social gain for the class, this enrichment for the 2nd-grade level was especially helpful for the exceptional children. For

[1] *Lessons in Arithmetic Through School Savings*, Elementary Grades, Activities, Problems, and Techniques, and *Teaching Mathematics Through School Savings*, Information, Activities, and Problems for Classes in Mathematics in Grades 7–9, by Irene M. Reid, Member, School Savings Committee, National Council of Teachers of Mathematics. Published by the Education Section of the U. S. Savings Bonds Division, Treasury Department, Washington 25, D. C. Free.

the class as a whole, experience in regular saving furnished motive power that carried all the pupils far beyond what they were expected to achieve at their grade level.

In *Lessons in Arithmetic Through School Savings*, activities and exercises using United States currency and Savings Stamps and Bonds are arranged approximately according to difficulty. Line drawings of U. S. coins and 10- and 25-cent Savings Stamps illustrate value relations of Savings Stamps and money. Problems in the purchase of Savings Stamps and Bonds present exercises in problems with and without numbers, as well as some in the graphic method of problem solving. A section of the study unit is devoted to facts about U. S. Savings Stamps and Bonds.

It is worth pointing out that in this arithmetic unit, the actual cost of the various denominations of Stamps and Bonds are the numbers used. In the study unit for grades 7–9, computations such as the percentage of student Stamp buyers in the room and in the school are real percentages the meaning of which is clear.

These simple and functional economics belong in that vast storehouse of information, skills, and understandings which children must acquire in their progress toward mature citizenship. There is now almost universal agreement among educators that children cannot begin too early to develop understandings about economic life. As J. A. Hockett[2] pointed out some years ago, "Elementary children are not merely preparing to be members of society; they *are* members" Many educators have expressed the conviction that the years of school are all too short to give these youngsters who are so soon to be workers, professionals, husbands, fathers, and citizens, an adequate preview of the economic problems they will meet and deal with in this important area of living.

Teaching Mathematics Through School Savings, for grades 7 through 9, extends the meanings of Stamp Day by offering teachers an opportunity to correlate actual experience in buying, saving, and investing with abstract theories of their subject. Especially in the seventh through the ninth grades, where percentage, interest, banking, savings, investment, insurance, and taxes make up a large part of the content of the mathematics course, student Stamp and Bond purchases provide a laboratory supplementing the textbooks. This is especially true where the study units are used and such problems as the interest paid on Savings Bonds and the interest paid on other types of securities are discussed.

These students are better able to understand current economic developments because of their School Savings experiences. Certainly those who are investors in U. S. Government securities are ready to learn more about the implications of their investment and its relation to the various aspects of the national economy. In presenting the statistics related to Stamp and Bond sales, the teacher can interpret national finance in a concrete form which students can readily understand.

The sales figures on Savings Stamps and Bonds add significance to the mathematical concept of the relation of the part to the whole. The student may see in the ascending totals in classroom, city, State, and Nation how the whole is compounded of such tiny parts as his own individual contribution, each essential to the completion of the whole. When he goes one step further to understand that these figures stand for ownership of the national debt, he sees how the burden of the debt, as well as its benefits in interest payments, are being spread widely among the people of America.

The integration of the School Savings Program with mathematics will enable students in the seventh through the ninth grades to approach the goals referred to in

[2] Hockett, John A., "Facing Realities in Elementary School Social Studies." *California Journal of Elementary Education*, Vol. 4, pp. 136–147, February, 1936.

14 of the 28 questions which compose the essentials for functional competence in mathematics on the checklist included in the second report of the Commission of Postwar Plans of the National Council of Teachers of Mathematics, published in the May 1945 issue of THE MATHEMATICS TEACHER.

The study unit also represents a list of mathematical concepts and suggested activities designed to help the teacher make mathematics more functional in the actual business of living.

In a section on the Payroll Savings Plan and another on the Bond-a-Month Plan, student understanding of these two important methods of saving is assured through the presentation of problems based on the outstanding features of these two plans for planned savings for adults.

Once he has identified his own interests with those of his community, and his own actions with national enterprise, the student has acquired several of the basic attitudes for good citizenship. In stressing the dependence of the whole upon the parts, the teacher can inspire the student to think of his savings as the fulfillment of a responsibility as an individual citizen. All Americans are equally responsible for keeping America strong, and all benefit from a strong, free country. From this sense of partnership can also come the understanding that the welfare of our country is both dependent upon and contributing to the welfare of the world.

Institute for Mathematics Teachers
University of Wisconsin

An Institute for Mathematics Teachers, sponsored by the School of Education and the Department of Mathematics, will be held at the University of Wisconsin July 20–22, 1948. Professor Ralph Beatley, Harvard University; Mr. William Betz, Rochester, New York; Miss Mary Potter, Racine, Wisconsin; Professor H. G. Ayre, Western Illinois State College; Professor Bjarne Ullsvik, Illinois State Normal University; and members of the University of Wisconsin faculty will address general sessions and lead discussion groups. In addition, many teachers of mathematics in Wisconsin secondary schools will appear on the program in panel discussions.

Among titles of scheduled addresses are: "The Impact of Mathematics on Method," "Teaching Mathematics Into Its Rightful Place," "Does the Taxpayer Get His Money's Worth in Arithmetical Competence in a High School Graduate?" and "The Basic Significance of Mathematics."

The theme of the Institute is "The Mathematics Teacher Looks at His Job." The program, organized to provide opportunity for discussion of teaching problems suggested by secondary school mathematics teachers, includes group and panel discussions on such topics as two-track program in mathematics, basic concepts in algebra and geometry, guidance in mathematics, measurement of understanding, the textbook, field work, the accelerated student, a 12th grade class for students deficient in essentials, and interesting stories for motivation.

An exhibit of classroom materials prepared by students, demonstration of visual aids, and visits to the U. S. Forest Products Laboratory and University Engineering Laboratories are also planned.

Rooms will be available in one of the University dormitories. For information or reservations, write to J. R. Mayor, North Hall, University of Wisconsin, Madison.

A Golden Decade of Popular Mathematics

By Daniel B. Lloyd
Calvin Coolidge High School, Washington, D. C.

Ten years ago the author published in a professional journal[1] his original "Bibliography of Popular Mathematics." The constant demand since then for reprints of this has attested to the need for such material by school clubs and individuals interested in the popular side of mathematics. The intervening decade has witnessed the publicatiou of an unprecedented amount of material devoted to interesting and popular phases of this subject, which throughout the ages has been a constant lure for the layman and the professional investigator alike.

The following bibliography[2] is confined to this intervening decade and includes accessible material published in English and available at schools and libraries. It is divided into two sections, (I) Books and (II) Periodicals, arranged topically thereunder. The field covered is that of elementary mathematics, for students and teachers, from intermediate grades through the junior college, as well as for the layman desiring to derive pleasure and culture from this classical subject. It will probably find its greatest use as a handy reference list of supplementary and enrichment material for motivating the study of mathematics in the nation's schools.

Most of the articles are readable by students and appropriate for presentation to their classmates, under teacher guidance. Some of the books contain more mature treatments and could be used advisedly as reference books. Listings are not alphabetical or chronological, but topical in the logical order studied, as far as possible, in conventional courses.

[1] See *School Science and Mathematics*, February, 1938.

[2] Copies of either the old or of this new Bibliography may be obtained for 25¢ each in stamps or coin from the author in Washington, D. C.

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II. ARTICLES

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A Mathematical Prodigy: History & Legend, (Galois), S.M., 5: 83–94.
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(See also "Measurement" and "Numbers")

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Arithmetic Devices, Nat. Elem. Principal, 22: 120–21.
Drill Devices in Arithmetic, School (Elem. Edition), 32: 327–29.
Equipment for Numbers, Instructor, 59: 20–22.
Arithmetic Can Be Fun, Texas Outlook, 28: 23–4.
Arithmetic Puzzles for Junior Grades, School (Elem. Ed.), 29: 809.
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Results of Recent National Council Elections

At the recent annual meeting of the National Council of Teachers of Mathematics in Indianapolis, the following elections were announced by the Board of Directors.

Honorary President—W. S. Schlauch, Dumont, N. J.
President—E. H. C. Hildebrandt, Northwestern University, Evanston, Ill.
First Vice President—H. W. Charlesworth, East High School, Denver, Colo.
Second Vice President—Vera Sanford, Teachers College, Oneonta, N. Y.

Directors for Three Years:
George E. Hawkins, Lyons Twp. High School, La Grange, Ill.
Marie S. Wilcox, Washington High School, Indianapolis, Ind.
James H. Zant, Oklahoma A and M College, Stillwater, Okla.

Miss Elenore M. Lazansky was chowen by the Board to replace Miss Emma Hesse (deceased) for the coming year. Edwin W. Schreiber was re-elected Secretary-Treasurer and W. D. Reeve was chosen as Editor of The Mathematics Teacher for a term of three years.—Editor

Notice to Teachers of Mathematics

Mrs. Ida Mae Heard of Southwestern Louisiana Institute at Lafayette, La. is making a collection of jokes, puns, witty sayings, boners, anecdotes and the like pertaining to mathematics. Her College plans to print her collection in July 1948, but she thought that some of the readers of The Mathematics Teacher might like to send some such items to her, the best of these to be included in her printed collection. The printed collection will then be made available to all teachers on a cost basis plus 15%.

Anyone having such material, please send it at once to Mrs. Heard.—Editor

Theory and Practice

An Assembly Program

By IRVING ADLER
Straubenmuller Textile High School, New York City

(AS THE curtain rises, a girl is talking with much animation. Her companion, a boy, is the picture of skepticism.)

GIRL: And that's my plan. What do you think of it?

BOY: There's nothing in it. It sounds alright in theory, but it won't work out in practice.

VOICE: (Coming from nowhere, everywhere, filling the room): Hold on there! That doesn't make sense!

(Boy and girl look around, surprised, bewildered.)

BOY: Wh-who are you?

VOICE: Who am I? People who think straight and talk sense have no trouble recognizing me. But you just spoke some nonsense.

BOY: (Belligerently) Whaddaya mean! All I said was that her plan may be good in theory, but it won't work out in practice!

VOICE: Precisely. You consider yourself very practical, don't you?

BOY: Sure! I don't go in for any theoretical stuff. I say experience is the best teacher.

GIRL: You can learn from books, too!

BOY: Huh! There she is with books again. Look. I learnt how to drive a car and I didn't learn it from a book, either. (Aggressively, advancing on girl, pointing finger into her face as she retreats.) Did you learn to drive a car from a book? No! Did anybody ever learn to drive a car from a book? No! See! What did I tell ya!

VOICE: We'll give you a chance to show how practical you really are. Bring up that board that you see in the corner.

BOY: Hey, this looks like a pin-ball machine.

GIRL: And here's a box of marbles. I see. You drop the marbles in at the top, and they collect in these chambers at the bottom.

BOY: Yeah. I can see that, too. But what's it got to do with me?

VOICE: There are 64 marbles in that box. If you drop them all in, one at a time, how many will collect in each chamber?

BOY: Aw, that's easy. Just count the chambers. One-two-three-four-five-six-seven. I'd say—let's see now—seven into sixty-four—that's about nine in each. See! Here, I'll show ya! (He starts dropping the marbles in. The girl interrupts.)

GIRL: Wait! I've been thinking—

BOY: (With disgust) What, again! Come on, let's not waste any time.

GIRL: I have it. You're all wrong. The distribution depends on chance and therefore won't be the same every time. But the approximate distribution will probably be one in the first chamber, six in the second chamber, then 15, 20, 15, 6 and 1. (Writes on the board as she speaks.)

BOY: You and your crazy ideas! (Resumes the dropping of the marbles until all 64 are in the chambers.)

VOICE: Now count the marbles in each chamber. (The boy counts and calls out the figures while the girl writes them on the board.)

VOICE: I'm afraid, my boy, that you're not as practical as you think. Your answer was all wrong. Your see, there are very few in the end chambers and many in the middle chambers, as predicted by your friend.

BOY: Well, it's not my fault. I never did this before. How can you know what will happen if you never had the experience?

VOICE: Perhaps your friend can answer

that since her numbers gave a better result. Tell us, Miss, how did you know what to expect?

GIRL: Oh, I had a theory. I'll explain it to you. (Draws a diagram at the board.) This is what the board looks like. After a marble comes through the first opening, which way can it move?

BOY: Well, it can go to the left or to the right.

GIRL: That's right. If it goes to the left it comes through this opening. If it goes to the right it comes through that one. Since both paths are equally likely, then, out of every two marbles, one will probably go left and one will go right. Now, a marble can reach this opening (points to first space on third line) only if it comes from here. (Points to first space on second line.) How can a marble reach the second space?

BOY: (Pointing) From here moving right or from there moving left.

GIRL: That makes two paths leading to the center space. And only one path leading to the third space. Now, every marble coming through the third line must move left or right, left or right, left or right. (Draws lines as she speaks). How many paths lead to this space?

BOY: Only one.

GIRL: To this space?

BOY: Three. One from here, and two from there.

GIRL: This space?

BOY: Three again. And the last is one.

GIRL: Do you think you can figure out the number of paths to the openings on the next line?

BOY: I think I see it. Each marble goes left or right. So one can come here. One and three is four. Three and three is six. Here's another four. And one.

GIRL: Now get the next two lines.

BOY: (Continues the computation out loud. After reading the final figures.) That's what you said before. Well, whaddaya know!

VOICE: That brings us back to the statement I objected to before. Do you remember what you said?

BOY: Yes. I said her plan was alright in theory but wouldn't work out in practice.

VOICE: After your experience with the pin-ball machine, can you see what was wrong with the statement?

BOY: Well, I suppose I didn't see how useful a good theory can be.

GIRL: That's right. By using my theory, I was able to figure out approximately what would happen.

VOICE: Precisely. A good theory gives you understanding. And understanding is essential for intelligent practice.

BOY: Does that mean that all theories are good?

VOICE: No. A good theory must be based on practice. An idea that has no foundation in experience will not bear any practical fruit. If a theory doesn't work out in practice then it's no good as a theory.

GIRL: Do you see what he means? Theory and practice cannot be separated. We need them both.

BOY: I get the point. I can see how it worked out with the pin-ball machine. But that's only a toy. Are there really cases where important practical things are based on a theory?

VOICE: Are there? Just listen.

(Two voices alternate using contrasting tones and tempos. As each theory is mentioned, a girl emerges from the left carrying an appropriate poster. At the mention of each practical achievement, a boy enters from the right.)

VOICES: Mathematicians, playing with theoretical lines on theoretical spheres, developed spherical trigonometry.

By using spherical trigonometry, the captain of a ship at sea can find his position within two miles.

Clerk Maxwell, physicist, predicted from his equations that energy could be broadcast as electromagnetic waves.

Today the powerful radio industry stands as a monument to the correctness of the Maxwell theory.

Albert Einstein obtained as a conse-

quence of his theory of relativity the mass-energy equivalence equation, E equals MC-square.

Today the ruins of Hiroshima and Nagasaki are grim reminders that yesterday's theory is the foundation for today's practice.

BOY: I think I understand. And I know who you are, too.

VOICE: Who am I?

BOY: You are the voice of Science.

CURTAIN

Posters:

1. Maxwell equations.
2. Diagram of a radio circuit.
3. Spherical Trigonometry diagram and the Cosine Law.

Diagram that is placed on the board:

```
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
. 1 . 6 .15 .20 .15 . 6 . 1 .
```

4. A ship at sea.
5. Montage of diagrams showing the bending of light, the relativity precession of an electron orbit, and the equation E equals MC-square.
6. Sketch of a bomb, labeled ATOMIC BOMB.

Notice to Members

ANY ONE who began his subscription to THE MATHEMATICS TEACHER with the October issue in 1947 is automatically a member of the National Council of Teachers of Mathematics until October, 1948. However, since no issues of the magazine are published in June, July, August and September, those who paid their dues in October of last year should send in their renewals before October, 1948 in order to save the Council inconvenience and loss of money. Costs of publication are rising and in order not to have to raise the price of the journal any further (the present dues are $3) we bespeak the cooperation of our members in being prompt in making renewals. In order to make sure that this matter was not overlooked by subscribers, THE MATHEMATICS TEACHER sent out cards to all members whose subscriptions expire in May (even though their membership runs to October). It has been almost impossible to plan for the October issue each year because members are so careless about renewing in time. Moreover, entirely too many members fail to renew at all even though two or three notices and a personal appeal from the Editor have been sent out. It is more important now than even before to "stick by the ship" if we are to weather the storms ahead and if the Council is to continue to do its work effectively. This year in particular we cannot afford to send the October issue to those whose dues have not been previously paid.

—EDITOR

Increase in Price of Yearbooks

The National Council is compelled to increase the price of the 15th and 16th and 18th yearbooks to $3 each, postpaid, effective September 1, 1948. The 19th and 20th yearbooks are also $3 each postpaid. We are sorry to have to increase our prices, but we hope that our members will all secure increases in salary for the coming year, so that our increases will not be burdensome. In the days ahead it is extremely important for every teacher of mathematics in the elementary and secondary schools to show his allegiance to The National Council by keeping up his membership in that organization. We know you will not fail us.—EDITOR

Need for Monographs Showing Vocational Implications of Academic Subjects*

VOCATIONAL guidance literature makes much of the point that school subjects constitute a favorable medium for enlightening youth regarding vocations. The traditional subjects, in adition to constituting a base of general education, are applied in many vocations. We frequently advocate that subject-matter teachers point out these applications, but we have done little to help them do so.

Pioneer efforts in this direction were made by Jesse B. Davis whose book, *Vocational and Moral Guidance*, published in 1914 provided teachers of English with exercises and projects which promoted pupils' thinking about vocational problems. Several textbooks on civics have also touched on vocational problems. Some colleges such as Hunter and the University of Chicago have issued pamphlets indicating vocational possibilities growing out of academic subjects. During the 1920's the National Research Council published a few such pamphlets. Included in this literature are also the Champaign Guidance Charts.

Teachers of various subjects who have caught the vision have pleaded for more materials that will help them in setting forth the vocational implications of their subjects. To meet this need in mathematics, the National Council of Teachers of Mathematics charged its Commission on Post-War Plans with the responsibility for assembling materials that would set forth the vocational implications of mathematics.

* Editorial by Professor Harry D. Kitson in the April 1948 issue of *Occupations*—the Vocational Guidance Journal. Quoted by permission of Professor Kitson—EDITOR.

The Commission's report has just appeared—a 24-page pamphlet entitled, Guidance Report of the Commission on Post-War Plans. It is distributed by THE MATHEMATICS TEACHER, 525 W. 120th Street, New York City 27. Price, $.25 each or $.10 in lots of 10 or more.

Under the heading "Mathematics Used by Trained Workers" are sections devoted to bookkeepers, clerical workers, craftsmen, farmers, nurses. There is a list of 76 "apprenticeable trades that require mathematics." There is a section on mathematics for professional workers such as statistician, surveyor, engineer, actuary, etc.; sections on women in mathematics and mathematical organizations.

A bibliography contains useful references.

In producing the report, the Commission obtained the assistance of the Occupational Information and Guidance Service of the U. S. Office of Education. Indeed the "paragraph of acknowledgements" credits Walter J. Greenleaf with having organized the materials and having written the first overall draft.

In order to insure that the pamphlet be intelligible to students, the Committee asked two young people to read and criticize the manuscript before publication—a pattern worthy of adoption by all producers of occupational monographs.

There is room for many full-length monographs of this type. The enthusiastic reception given to this pamphlet by teachers of mathematics indicates that similar publications in the sciences, modern languages, fine and industrial arts, etc., would find a ready market.

Guidance Pamphlet in Mathematics

Have you ordered your supply of guidance pamphlets in Mathematics? We have sold over 20,000 to date. Price, 10¢ each in lots of 10 or more. Single copies, 25¢ each postpaid.

AIDS TO TEACHING

By

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BOOKLETS

B. 7—*Men of Science*
Westinghouse Electric Corporation, 306 Fourth Avenue; Box 1017, Pittsburgh 30, Pennsylvania
Booklet; 6″×9″, 45 pages; free.

Description: The fifteen stories of science are mostly biographical stories of men who have pioneered in research. Originally all of them were told as stories on the Sunday Westinghouse Radio Program by John Nesbitt. The subjects range from alchemy to television, including chemistry, astronomy, aviation, and agriculture among others; only one story concerns mathematics: "Man Is an Explorer, Albert Einstein and the Theory of Relativity." There are attractive drawings throughout the booklet and not much advertising.

Appraisal: The stories are delightfully written in the "wide-eyed" style of science reporting that seems in constant, breathless amazement over the achievements of scientists. In the story on Einstein there are a few human interest details without much attempt at explaining the meaning or importance of his work. The end of the story brings in a far-fetched illustration of how some of the work at Westinghouse is as pioneering as Einstein's. Nevertheless, this type of story may well serve as a model for short compositions developed jointly by the mathematics and English departments on scientific subjects.

B. 8—*Navigation* by Byrd H. Granger and M. L. Colbert
Air Age Research, 80 East 42nd Street, New York, N. Y.
Booklet; 6″×9¼″; 32 pages; free.

Description: This booklet is most useful when combined with the chart on navigation produced by the same company (See C.1.) In fact, the illustrations are the same as those on parts of the chart. There are chapters on the science of navigation, map projections, aeronautical charts, instruments, the course and the wind, and contact flying by night.

Appraisal: The material covered is excellent and well chosen for a high school level. There is a slight bit of historical material, and the constant feeling in the writing that the applications of mathematics and science are really practical, not artificial. The illustrations are simplified to the bare details and therefore very clear. However, there are not enough illustrations to explain the material sufficiently. For example, there are only two pictures to explain the whole idea of map projections, only two on aeronautical charts. As a beginning in geometry and trigonometry classes, however, the booklet should be very useful.

CHART

C. 3—*Biggest and Littlest Things in the Universe*
School Service, Westinghouse Electric Corp., 306 Fourth Avenue; P. O. Box 1017, Pittsburgh 30, Pa.
Chart; 24″×37″; $1.00 (or free with order of $5.00 or more)

Description: This colored chart is mounted on wooden sticks at top and bottom so that it can easily be rolled and stored. It portrays in pictures (not to scale) and in figures the relative sizes of twenty different measured things "from the universe itself to the proton, smallest particle of matter." There is a short descriptive paragraph about each picture.

Appraisal: The greatest use for this chart in mathematics classes will be to teach the "scientific" or exponential form of very large and very small numbers. Exercises in finding the ratios between various objects from different parts of the chart will give a great deal of practice and lead to plenty of discussion concerning multiplication and division of numbers with exponents, and the need for converting both numbers to the same units before finding their ratio. Science classes will find even more use for this chart than the mathematics classes. The paper is durable, the colors attractive, the material useful and the wooden rods will probably triple the life of the chart.

EQUIPMENT

E. 4—*Sterling Draft-Kit*

Sterling-Freeland Industries, Inc., 122 South Michigan Ave., Chicago 3, Illinois. Portable Drafting board and instruments; for $8\frac{1}{2}'' \times 11''$ paper, \$16.00; $11'' \times 17''$ paper, \$24.00 (Educational discounts.)

Description: These portable drawing boards are very well made. They have a pressed board base on which is mounted a beveled ruler along the left and bottom edges which serves as a guide and stop for the drawing paper. This paper is kept in place with a screwclip along the top edge. The imitation leather cover which folds over the kit from the left has a pocket which holds the transparent L-shaped substitute for a T-square, and the 45–60–75° triangle that are included. This triangle has a quadrant cut out of the center and is marked so that it can be used as a protractor.

Appraisal: The sturdy construction and small size of these boards should make them very useful in mathematics teaching. A large number can be stored in a very small space so that accurate, geometric drawings could be made as inductive introductions to plane-geometry concepts. Solid geometry classes might learn a great deal about perspective by making some careful drawings; the same classes would need drawing boards in planning lay-outs to construct solid models. Outside the classroom such portable boards would facilitate field sketching of surveying problems which could then be brought back into the classroom for computation.

The equipment is expensive for many schools but, in quantity, the discount might make it reasonable. The board and attachments are very strong and should last indefinitely; the cover is too thin and weak to last more than a few years of constant use. In particular, projections on the base will tend to break and wear through the cover unless care is taken. All-in-all, though, well worth a try.

FILMS

F. 12—*Introduction to Fractions*

F. 13—*How to Add Fractions*

F. 14—*How to Subtract Fractions*

Johnson Hunt Productions, 1133 North Highland Ave., Hollywood 38, Calif. 16 mm. sound film; each title 1 reel; black and white—\$45.00; color—\$85.00; 1947. Filmstrip; 35 mm. 1 filmstrip for each title; black and white—\$2.50; color—\$4.50; 1947.

Content of Introduction to Fractions: By animation of familiar objects such as a cake, apple, chocolate bars, disk, etc., this film defines and illustrates practical applications of fractions, numerator, denominator, improper fractions and mixed numbers. Illustrating with segments of a disk, numerator and denominator are compared and changed to show how the value of a fraction changes when the numerator or denominator changes in value. Several simple problems in determining the value of a fractional part are

presented and developed step by step. In the last problem, the answer is left for the pupil to determine.

Content of How to Add Fractions: Again, animation of familiar objects is used to demonstrate the part fractions play in daily life and the meaning of a fraction. Segments of a disk are used to review the meaning of numerator and denominator and to show why it is necessary to have both terms of a fraction to determine the value of a fraction. The addition of fractions, improper fractions, and mixed numbers which have a common denominator is illustrated with a very clear explanation of why the denominator does not change and why a common denominator is necessary in addition. Simple problems are used to show the changing of fractions to equal fractions in order to add fractions with unlike denominators.

Content of How to Subtract Fractions: This film opens by reviewing the definition of a fraction, numerator and denominator. The meaning of subtraction is illustrated for such problems as $\frac{5}{8}-\frac{3}{8}$, by showing a pie divided into eight parts and pieces of pie taken from the plate. The film explains why a common denominator is necessary in subtraction and why the denominator does not change. Simple problems on the subtraction of fractions with like denominators, subtraction of a fraction from a whole number or a mixed number, and subtraction of fractions which do not have a common denominator are explained step by step and illustrated by real problems such as the purchase of one-half pound of butter from a supply of three and one-fourths pounds.

Appraisal of F. 12, F. 13, and F. 14: By means of three-dimensional animation, these films should assist the child in obtaining a visualization of the physical significance of fractions. They are designed primarily to aid the teacher in clarifying and summarizing concepts and processes which have been presented in classroom instruction. Although the material moves too rapidly for even an alert child to follow in detail without previous classroom preparation, it could properly be used as a review and orientation to the subject for beginning classes or for reviews and remedial work at higher levels. These films will also be useful in the training of arithmetic teachers since they show how to make abstract concepts meaningful. The absence of live characters in the picture is an effective means of focusing the attention of the student to proper place. The films give excellent exposition of the meaning of a fraction and how its value is related to the numerator and denominator.

These three films are the first three of an integrated series of eight motion pictures in fractions being filmed in color to increase the attractiveness of what is too often an unattractive subject. Black and white prints are also available as well as filmstrips which contain 35 key scenes from the picture to which it relates. A teacher guide is also available for each film.

In the opinion of this reviewer these are some of the best films produced on mathematics.

Technical Qualities: Photography: Excellent. Content: Excellent. Sound: Excellent.

F. 15—*Parts of Things*

Young America Films Inc., 18 East 41st Street., N. Y. C.

Technical Advisors: Wm. A. Brownell and Laura K. Eads

16 mm. sound film; 1 reel; black and white—$38.50, 1947.

Content: This film introduces the student to the meaning of halves and fourths through the medium of simple line-drawing animation. Two boys divide items such as apples, melons, a quart of milk into two or four equal parts. These objects are then used to show that there are two halves in an object and that the two halves are equal to each other and that the symbol $\frac{1}{2}$ means one of two equal parts. Fourths are discussed in a similar fashion. The film ends by showing the fractions used in a muffin recipe.

Appraisal: This film develops elementary concepts that are highly important for an understanding of fractions. It provides a gradual development from concrete thinking to simple abstractions, to more complicated abstractions involving symbols and the application of fractions in practical problems. The commentary includes many questions to stimulate thinking. It is commendable that it presents few concepts. It deals only with two fractional parts of objects, leaving fractional parts of groups for later development. However, it is doubtful that this film will be as effective as the use of the concrete objects by a skillful teacher. The superposition of objects to show equality will probably not be understood by primary pupils. Attention may frequently be focused on the boys in the pictures rather than on the objects illustrating the fractions. Includes teachers guide.

Technical Qualities: Photography: Good drawings. Commentary: Very clear and appropriate for elementary pupils. Content: Satisfactory. Level: Grades 2–4.

FILM-STRIPS

FS. 4-FS. 19—*Plane Geometry*
Curriculum Films, Inc., R.K.O. Building, Radio City, N. Y. C.
Film-strip: 35 mm; color—$50.00 for set of 16; $4.95 each film strip.

FS. 4—*Introduction to Plane Geometry.* 43 frames

Defines geometry as a science that deals with relationships, constructions, descriptions and measurements of figures that can be drawn on flat surfaces. Defines and illustrates point, line, and plane. Illustrates shape, size, and relationships: geometrically and otherwise. Stresses the logical basis for all geometric proof and illustrates the need for good reasoning in mathematics, science, and aviation.

FS. 5—*Vocabulary: Lines and Angles I*

FS. 6—*Vocabulary: Lines and Angles II, Relationships.* 50 frames

FS. 7—*Vocabulary: Lines, Relationship.* 23 frames

Define and illustrate different kinds of lines and angles such as line segment, parallel lines, transversal, perpendicular lines, vertical, oblique, and horizontal lines, acute angles, adjacent angles, vertical angles, complementary and supplementary angles. Illustrate how lines and angles are formed, rotation, parts, and measurement of angles. Describe the conditions for parallelity. Each film-strip contains brief, informal exercises for review and identification.

FS. 8—*Vocabulary: Triangles.* 39 frames

Defines, illustrates and compares different kinds of triangles. Describes the parts of triangles including exterior angles and the sides of right triangles. Illustrates the special lines in triangles; namely, medians, angle bisectors, and altitudes.

FS. 9—*Vocabulary: Polygons.* 56 frames

Defines and illustrates polygons of different size and shape classified according to sides and angles. Shows poloygons of 3, 4, 5, 6, 7, and 8 sides, convex and concave. Gives examples of regular polygons, trapezoids, parallelograms, and the quadrilateral "family tree." Includes eight frames for identification practice and review.

FS. 10—*Vocabulary: Circles I.* 35 frames

FS. 11—*Vocabulary: Circles II.* 23 frames

Define and illustrate the circle and all the common terms connected with a circle such as radius, diameter, tangent, secant, chord, arc, semi-circle, segments, sector, concentric circles and circumscribed polygons. These terms are vividly illustrated by a pirate sharpening his sword on an emery wheel. In order to focus attention on the term illustrated, the tangent and point of tangency are labeled. The kinds of angles formed in circles and their measurement are also illustrated. Many drawings are included with questions to

assist the student in review and identification.

FS. 12—*Postulates: Lines.* 56 frames

FS. 13—*Postulates: Triangles and Circles.* 34 frames

Defines and illustrates the following postulates by everyday examples showing how we can assume them to be true without proof: lines can be extended infinitely far; one and only one straight line can be drawn through two points; a straight line is the shortest distance between two points; two lines intersect in only one point; every line has only one midpoint; geometric figures can be moved without changing their shape; only one perpendicular can be drawn at a point on a line; the perpendicular distance is the shortest distance from a point to a line; only one line parallel to a given line can be drawn through a given point; corresponding parts of congruent triangles are equal; with a given radius, one and only one circle can be drawn; equal circles have equal radii; radii of equal circles are equal; circles with equal radii are equal in area; a straight line can intersect a circle in not more than two points.

FS. 14—*Locus.* 61 frames

Defines locus as the path of things moved according to certain conditions. The five basic loci are described geometrically and illustrated by apt examples from everyday life. For example, the locus of a point equidistant from two points is illustrated by a stone being shot from a sling shot.

FS. 15—*Geometry in Art.* 57 frames

By means of beautiful color pictures this filmstrip shows applications of geometry to everyday life illustrating how simple geometric forms are used in home decoration, church windows, cubist, abstract and non-objective art, abstract motion, pictures, advertising and modern, ancient and oriental architecture. Illustrates the place of symmetry in architecture, wrought iron work, churches and homes.

FS. 16—*Logic: Definitions and Key Words.* 42 frames

FS. 17—*Logic: Deductive Reasoning.* 24 frames

FS. 18—*Logic: Induction, Analysis, Indirect Reasoning.* 46 frames

FS. 19—*Logic: Mistakes in Thinking*

These filmstrips diagram the steps to follow in logical reasoning and illustrate these steps in geometrical and in non-geometrical situations. The filmstrip on definitions and key words stresses the importance of precise formulation of definitions to support reasoning and draws attention to the key words, "any," "all," "no," "none," and "some" in definitions. The elements of deductive reasoning illustrated are: (1) general statement, (2) specific statement, (3) specific conclusion. The definitions of induction, analysis, and indirect reasoning are illustrated by examples from the reasoning of scientists, detectives, mechanics, and high school students. The mistakes in thinking which high school students often make in daily life, namely, hasty generalization, prejudiced conclusion, and non sequitur, are illustrated by appropriate student problems such as dating.

Appraisal of FS. 4–FS. 19: This series of geometry filmstrips is the best of the mathematics film strips that the writer has viewed. There are several reasons for this high rating. For one thing the pictures are beautiful Americolor. This color has been used as a teaching device by making equal angles and lines the same color and to accentuate geometric relationships. Sometimes the color contrasts are not great enough to make important parts of illustrations stand out, but the color combinations are always attractive. Secondly the filmstrips contain a great number of apt applications to everyday life. In addition, simple exercises to assist

learning are included. The series is also accompanied by a teacher's manual that will give much assistance to the teacher using these aids. These filmstrips can easily be used to enrich and supplement the usual geometry course since each filmstrip is integrated with the present curriculum. The four filmstrips in "Applying Geometric Logic" should make it possible for teachers to attain the objective of better habits of reasoning and an appreciation of the use of these methods of reasoning in everyday life. It is unfortunate that the filmstrips are entirely drawings rather than actual photographs.

Technical Qualities of FS. 4–FS. 19: Photography: Good drawings in color. Content: Excellent. Level: Junior and senior high school. Many of the filmstrips are usable in the intuitive geometry of the junior high school. (Reviewed by D. A. J.)

MODEL

M. 1—*Multi-Model Geometric Construction Set*
Yoder Instrument Co., East Palestine, Ohio
Model of solid geometry exercises—$35.00.

Description: This construction set is made up of plates, grills, rods, and elastic chords that can be used to construct a variety of models. These models can be used to illustrate theorems, postulates, propositions, and corollaries of plane and solid geometry, trigonometry, mechanical drawing, and descriptive and projective geometry. The plates and grills are used to represent planes while the rods and elastic chords represent lines. Plane figures can be illustrated by using one plate for a plane and elastic chords as lines. The plates, rods, and chords can be joined in a variety of ways to illustrate most of the relations between lines and planes, parallel and nonparallel planes, solids and planes that are studies in solid geometry. The building of the models is so easy that the instructor or students can set them up very quickly.

Appraisal: This device should make it possible for every solid geometry student to "see" two-dimensional drawings of solids and to understand better the relations between lines, planes, and solids. As a piece of equipment for the mathematic's classroom that the student can manipulate, it should add interest and meaning to one phase of mathematics instruction.

SOURCES OF MATERIAL FOR LABORATORY WORK

SL. 3—*Ditto Workbooks*
Ditto, Incorporated, Harrison and Oakley Blvd., Chicago 12, Illinois
Arithmetic and algebra workbooks for reproduction; $8\frac{1}{2}'' \times 11''$; see below for description and prices.

Description: The pages of the Ditto Workbooks are master copies from which the schools can make as many copies as are needed, by either of two ditto reproducing methods: the gelatin and the liquid. One hundred clear copies can be made by the former process, up to 300 copies by the latter. Changes can be made in the material to suit individual needs if necessary, or new pages added. For this purpose various special pencils, typewriter ribbons, carbon paper, and inks can be secured from the company. As many as eight different colors can be used on a single sheet.

The following workbooks are available:

Gelatin Process:

Arithmetic: Fifteen workbooks, one for grade 1, and two each for grades 2 through 8 (one for each half of year). Most books have tests included. $2.00 each.

Algebra: One workbook for first year algebra. Contains diagrams, graphs, and many opportunities for pupil work other than mere computation or problems. $2.50.

Liquid Type Process:

Arithmetic: First three of series above (Grades 1 and 2) $2.75 each.

Appraisal: These workbooks are filled with appropriate and useful materials. They seem to be designed by authors who know what is needed for workbook material. Using them is certainly much less expensive than buying complete workbooks for each pupil, but the initial cost and up-

keep of the reproducing machines must be figured as well as the cost of paper and ink before a fair comparison can be made. Moreover, there is the labor of the individual who runs off the copies. The greatest advantage over complete workbooks is the ease with which pages can be omitted, added or changed. The chief criticism of the material included is that it is predominantly tied to the older type of drill on skills and processes, without enough examples of applications and opportunity for original initiative. This is probably inevitable when an attempt is made to fit a workbook to a large number of different curricula: only the skeleton remains and that cannot live by itself.

Notice to Readers of The Mathematics Teacher

Please do not send money, stamps, or checks to THE MATHEMATICS TEACHER in payment for any of the National Council Yearbooks. All such orders and payments for the same should be sent to *The Bureau of Publications*, Teachers College, Columbia University, 525 W. 120th St., New York 27, N.Y.—EDITOR

◆ THE ART OF TEACHING ◆

Teaching Mathematics in Technicolor

By LAURA BLANK
Hughes High School, Cincinnati, Ohio

THE use of color as functional, as an interest value and, to a small extent, as aesthetic in the teaching of abstract mathematics is probably to most instructors a novel idea. Year after year pupils have appreciated experience with it in work on black board, bulletin board, paper and with models. It is possible that publishers, at some future time, may find it not too expensive to use color in purely abstract work in text books on mathematics.

Color has been used, of course, for sheer interest, long ago in concrete matter in the texts on elementary mathematics. Three gay red apples are more attractive than three drawn in black and white, for after all, who ever saw a black and white apple fit for pupil or teacher to eat? Children are, for the most part, realists.

But one day, much further along in our young friend's study of mathematics, one wishes to teach the commutative law of addition and multiplication. After illustrating with small groups of concrete objects, one might write $2+9=11$, then just below, $9+2=11$, writing each of the 2's in red chalk, each of the 9's in green, the rest of the work being in the conventional white, if one is using a black board. Again, $7\times 6=42$ then just below it $6\times 7=42$, writing each of the 6's in blue chalk, and each 7 in yellow, the rest of the illustration in the conventional white. In the later analogous literal work in algebra explained and written similarly one would possibly refer to this as an illustration of the operation of the "commutative law" of addition and multiplication. One would likely emphatically note in this connection, with illustrations, that this law of mathematics is invalid in subtraction and division.

In algebra if it is the wish to teach the expansion of $(x+y)^2=x^2+2xy+y^2$, the plus sign of the y in the original statement and the plus sign of the $2xy$ in the completed statement might well be stressed by writing them in red whereas the rest of the work would be in white on black board or black on white paper. Then, in contrast, the corresponding signs of the expression $(x-y)^2=x^2-2xy+y^2$ might be picked out in green, or the red again, writing the latter statement exactly beneath the corresponding former assertion, sign below analogous sign.

In trigonometry one can extend such a procedure one step further, after first stating: $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and $\sin(A-B)=\sin A\cos B-\cos A\sin B$, using contrasting colors in writing those signs only where emphasis of contrast is desired; then summarizing: $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$, here using, as above, red in the case of the plus symbols and green in the case of the negative signs.

Again in the algebraic field in the case of an application of the Binomial Theorem: $(r+s)^5=r^5+5r^4s+10r^3s^2+10r^2s^3+5rs^4+s^5$, write the exponents of "r" in one color, whereas those of "s" are in a different color, the rest of the work being in the conventional black and white.

$\text{Log}_{10}\ 1000=3$. Exponentially, $10^3=1000$, write the 10's in one color, the 3's in a second, the 1000's in a third for the association and the contrast desired.

Suppose the word "equation" has been misspelled by a pupil. Write it, with the "t" standing out in red chalk, crayon or

ink. Any word frequently spelled incorrectly can be written large on the board, those letters only in color where the error occurs, no reference being made to the specific error but only to the fact that one sometimes is made. In other words, correction is positive always, if it is possible.

x^2-y^2 factors into $(x+y)$ $(x-y)$; $4a^2-9b^2$ has for its factors $(2a+3b)$ $(2a-3b)$. Stress the plus and minus signs of the factors in color.

$X^3-y^3=(x-y)(x^2+xy+y^2)$. Write the two minus signs and the plus sign of the term $+xy$ in the same color. Whereas: $x^3+y^3=(x+y)$ (x^2-xy+y^2). Here color, probably with a color different from that used in the example above, the corresponding signs, that is of the $+y^3$, $+y$, and $-xy$. Finally one can summarize, as in the case from trigonometry above, $(x^3\pm y^3)=(x\pm y)$ $(x^2\mp xy+y^2)$, using two colors, one for the three terms of especial interest in the sum expansion, the other for the corresponding three terms of the difference expansion.

Write: $(\quad)^2-(\quad)^2$, the exponents and negative sign being in color. Then insert in each parenthesis a single term, literal or arithmetic. Factor, stressing with the same color the plus and minus sign of the two factors. Then, just below, write $(\quad)^2-(\quad)^2$, again, the exponents and negative sign being of the same color as in the preceding example. Insert a binomial and a monomial. Factor, coloring plus and minus signs and retaining the parentheses, for the moment. In a last step remove them. Again, just below, write $(\quad)^2-(\quad)^2$, coloring the exponents and sign with the same color. Insert a binomial in each parentheses. Factor, retaining parentheses, using the same color for the plus and minus sign between the pairs of parentheses. Finally remove the parentheses.

We have in algebra, let us say, a verbal problem to solve involving a rectangle. Its base is to be increased, its altitude, either increased or decreased. Draw the original rectangle conventionally. Elongate the base and appropriately change the altitude, in color. Finally completely outline this new rectangle in the same color. In other words, make the solution a "chalk talk" in color, as far as the diagram goes. Yes, even in building up the statements of the solution, those involving the original rectangle may be written conventionally, white on black, whereas those applying to the revised rectangle, written in its own color. An ambidextrous pupil with white chalk in one hand and green in the other will put on a most interesting demonstration, in such a case.

Now in geometry, write that portion of the theorem which contains the facts of the hypothesis in one color whereas the part which contains the facts to be proved in another. Draw an appropriate diagram. One usually is needed. In the corresponding color write the hypothesis in terms of the letters of this particular figure, and in the other color the specific conclusion in terms of the particular figure. Among the statements and reasons of the demonstration, those coming directly from the hypothesis can be written in its color. The conclusion when reached may be written in its color.

Lines introduced in the course of a proof may be drawn in color as well as dashed for emphasis. In an indirect proof, lines that are regarded tentatively as intersecting or on the other hand as being nonconcurrent, may be drawn in color, the better to differentiate them from those of the hypothesis.

The three medians of a triangle are to be constructed. Draw all of the arcs and the lines necessary for each in its own color, using chalk, crayons, or blue ink with black ink with red ink. A line segment is to be divided in four equal parts. Draw each bisection in its entirety in one color.

In the theory of proportions, we are interested in the extremes. Write conventionally: $\frac{\quad}{\quad}=\frac{\quad}{\quad}$, also $\ :\ =\ :\ $. Then insert, in one color, "r" and "s": $\frac{r}{\quad}=\frac{\quad}{s}$, also

$r : = : s$. Then in a contrasting color insert the two means to complete the proportion.

In solid geometry a plane intersects another or several other planes. In a model, make each plane of a different color of cardboard; in a drawing, each of a different color of chalk, the intersection of one with any one plane a dashed line of the same color as the plane intersected.

Two spheres intersect. Let their intersection be in color. One is interested in map projections. Consider a cone as intersecting a sphere, or on the other hand being tangent to it, show the sections in contrasting colors. Or, as in the case of the Mercator projection used by the navy, the grid of the meridians and parallels as we assume them on our earth are projected on a circular cylinder tangent to the sphere. Show the parallels and the projection of the parallels in one color, and the meridians and their projection in another.

The number of functional uses of color in the teaching and study of mathematics is infinite. The examples suggested here are few. One could rewrite any accepted text constructively. Pupils quickly learn its use and respond with pleasure in its application. Their ingenuity in the matter is truly self expressive.

Moreover as has been suggested, it lends life to an otherwise drab subject. Surely one would never think of mathematics as gay. But to anticipate a color and not know what one, does tend to keep one on the qui vive. The instructor finds herself guessing what color will appeal to Jack and what to Joan, and if the instructor why not the pupils? Moreover one can use colors appropriate to the time of year and its holidays.

There is a caution however. That is that not all colors are equally visible on a black board from the rear of a room. Some of the loveliest shades and tints must be reluctantly put aside for some other use in the art department of the school. Then, too, learn to associate colors well. Clashing colors defeat the end in that all interest is centered in the color conflict, for those pupils who are aware of it. Moreover if we are to teach aesthetic values in one department of our school we should, within reason do so elsewhere. Pupils who enjoy good color combinations or contrasts, by subdued exclamations show their mild appreciation. The reverse statement is also true.

We are learning to use color scientifically. We are tinting the walls and ceilings of our school rooms so as to make a more pleasant environment. We are experimenting so as to know which colors rest and relax us and which stimulate and excite us. We find them used for emphasis to catch our attention in the advertisements of our daily papers even. If it pays to run a daily paper a second time through the presses to catch and hold our interest with color, then we should catch and hold the interest of pupils in their educational work with color. If I may be permitted to become personal, one of my pupils characterized this technique of mine as "mathematics in technicolor."

Solicited

Mathematics Teachers in several states have established The Teachers' Collection of Trigonometries in order to preserve American published Trigonometries. Reports are sent out so that Contributors may know who contributed what. There are still many texts which are not in the Collection. Send lists of your possible contributions to Norman E. Dodson, Head, Math. Dept., Lenoir Rhyne College, Hickory, N. C. Selections will be made from lists received.

Mathematics Institute for Teachers

Duke University, Durham, N. C., August 9–20, 1948

***Director:* Professor W. W. Rankin, Duke University**
***Assistant Director:* Miss Veryl Schult, Washington, D. C.**

"Mathematics at Work" will be the general theme of the Institute this year. The motto of the Institute is "There is a Simple and Pleasant way to Study Mathematics."

Last year 170 mathematics teachers from 33 states attended. This year the registration will be limited to 200.

For detailed program and other information write Professor W. W. Rankin, Duke University, Durham, N. C.

PROGRAM

August 9, Monday

8:00 P.M.

Presiding: Professor C. R. Vail, Engineering, Duke University
Organization of Study Groups
Miss Veryl Schult

"Dynamic Stability of Guided Missiles"
Dr. F. G. Myers, Engineering Division, Glenn L. Martin Co., Baltimore, Md.

Discussion

August 10, Tuesday

10:45 A.M.

Presiding: Lt. Col. R. C. Yates, Mathematics, U. S. Military Academy
"Mother Nature is a Mathematician"
Dr. Walter H. Carnahan, Editor, Mathematics Dep't, D. C. Heath Co., Boston, Mass.
"Palatable Mathematics"
Professor W. W. Rankin, Mathematics, Duke University

Discussion

6:30 P.M.

Banquet—Union Ball Room—West Campus (informal)
Toastmaster: Professor W. W. Rankin, Duke University
Address: "General Education in a Technical Society"
Dr. Dwayne Orton, Educational Director, International Business Machines Corporation, New York, N. Y.

Social Hour

August 11, Wednesday

10:45 A.M.

Presiding: Miss Janet S. Height, Manchester, Mass.
"The Non-Academic Student in Mathematics"
Miss Mary A. Potter, Supervisor Mathematics, Racine Wis., Former President, National Council of Teachers of Mathematics

"A Study of Some of the Parameters Affecting Rocket Performance"
Dr. C. H. Harry, Engineering Division Division, Glen L. Martin Co., Baltimore, Md.

Discussion

8:00 P.M.

Presiding: Professor W. A. Gager, Mathematics, University Florida
"Applications of the Exponential Function"
Professor J. W. Cell, Mathematics, North Carolina State College

Discussion

August 12, Thursday

10:30 A.M.

President: Professor H. F. Munch, Education, University, N. C.
"Some Properties and Applications of Plane Curves"
Lt. Col. R. C. Yates, Mathematics, U. S. Military Academy

Discussion

8:00 P.M.

Presiding: Professor W. M. Nielsen, Physics, Duke University
"Design and Use of an Infrared Spectrophotometer for the Analysis of Organic Mixtures"
Dr. E. J. Martin, General Motors Research Laboratories, Physics Instrumentation Department, Detroit, Mich.

Discussion

August 13, Friday

11:00 A.M.

Presiding: Mr. Gaylord C. Montgomery, Clayton, Mo.
"Mathematics at the Proving Grounds"
Mr. Kenneth Stonex, General Motors Proving Grounds, Mich.
Discussion

8:30 P.M.

Reception: University House, Chapel Hill Street
Presiding: Dr. Clyde A. Erwin, State Supt. Education, North Carolina
Address: "Mathematics in Aeronautics"
Admiral Alfred M. Pride, Chief of Bureau of Aeronautics, U. S. Navy, Washington, D. C.

August 14, Saturday

10:30 A.M.

Presiding: Professor W. J. Seeley, Engineering, Duke University
"Automatic Digital Computing Machines; Their Present Present and Future"
Dr. E. W. Cannon, Head Machine Development Laboratory, National Applied Mathematics Laboratories, Bureau of Standards, Washington, D. C.
"The Mathematics of Aeria 1 Mapping"
Mr. G. C. Tewinkel, Mathematician, Coast and Geodetic Survey, Washington, D. C.

Discussion

6:30 P.M.

Dinner: Union Ball Room, West Campus
Toastmaster: Professor W. H. H. Cowles, Mathematics, Pratt Institute, Brooklyn, N. Y.
Address: "Mathematics Comes into the Laboratory"
Mr. Everett S. Lee, Chief Engineer, General Engineering and Consulting Laboratory, General Electric Co., Schenectady, N. Y.

Social Hour

August 15, Sunday

11:00 A.M.

Duke University Chapel Service

12:10–12:30 P.M.

Organ Recital
Mrs. Mildred L. Hendrix, Organist, Duke Chapel

4:00 P.M.

Carillon Recital
Mr. Anton Brees, Carillonneuer, Duke University

4:30 P.M.

Tea—Rankin Home, 1011 Gloria Avenue

August 16, Monday

10:45 A.M.

Presiding: Miss Hariett Doheny, Mathematics, Seattle, Wash.
"Mathematics Can Be Fun"
Miss Veryl Schult, Director of Mathematics, City Schools, Washington, D. C.
"Stratagems in Mathematics and Machine Design"
Mr. Sigmund Rappaport, Consulting Engineer, Wright Automatic Machinery Co., Durham, N. C.

Discussion

8:00 P.M.

Presiding: Professor Helen Barton, Mathematics, Womans College University of N. C.
"Mathematics in Quality Control"
Mr. D. K. Briggs, Sound Instrument Engineer, Western Electric Co., Burlington, N. C.

Discussion

August 17, Tuesday

11:00 A.M.

Presiding: Miss Bonnie Cone, Mathematics, Charlotte, N. C.
"Mathematics of Mechanisms"
Mr. John W. May, Chief Engineer, Wright Automatic Machinery Co., Durham, N. C.

Discussion

8:00 P.M.

Presiding: Miss Ida M. Bernhard, Mathematics, Southwest Texas State College
"Applications of Mathematics to Geodetic Surveys"

Mr. Lansing G. Simons, Coast and Geodetic Survey, Washington, D. C.

Discussion

August 18, Wednesday

11:00 A.M.

Presiding: Professor Sheldon C. Myers, Mathematics, Western Michigan College, Kalamazoo Mich.
"Some Elementary Geometry Constructions"
Professor J. M. Thomas, Research Mathematician, Duke University

Discussion

6:30 P.M.

Dinner: Union Ball Room, West Campus
Toastmaster: Professor Ruth Stokes, Mathematics, Syracuse University
Address: "Calendar Reform and the World Calendar"
Mr. Westy Egmont, Director General World Calendar Association; Editor of the *Journal of Calendar Reform*, New York, N. Y.

Discussion

Social Hour

August 19, Thursday

11:00 A.M.

Presiding: Mrs. Nannete Blackiston, Supervisor of Junior H. S. Mathematics, Baltimore, Md.
"Mathematics in Communication Research"
Dr. W. H. Bode, Research Mathematician, Bell Telephone Laboratories, Murray Hill, N. J.

Discussion

8:00 P.M.

Presiding: Dr. Catherine Lyons, Mathematics, Pittsburgh, Pa.
"The Mathematical Side of Actuarial Work"
Mr. Edward H. Wells, Actuary, The Mutual Life Insurance Co. New York, N. Y.

Discussion

9:30 P.M.

Party—Rankin Home, 1011 Gloria, Avenue (Watermelons)

August 20, Friday

9:00 A.M.

Presiding: Professor Miles C. Hartley, Mathematics, University of Illinois
"The Arithmetic of Lightning"
Professor C. R. Vail, Electrical Engineering, Duke University

Study Group Reports
Committee Reports

10:30 A.M.

ADJOURNMENT

STUDY GROUPS

I—8:00–9:00 Daily

Aids in the Study of Algebra (Junior and Senior High School Level)
Leader: Miss Frances Johnson, Mathematics, Oneonta High School, Oneonta, N. Y.

II—8:00–10:30 A.M. and 2:30–5:00 P.M. Daily

Laboratory in Mathematics (Construction and use of Models and Instruments for high school and college mathematics)
Leader: Lt. Col. R. C. Yates, Mathematics, U. S. Military Academy, West Point

III—8:00–9:30 A.M. Daily

Field Work, Surveying, Mapping, etc. (High School Level)
Leader: Professor H. C. Bird, Civil Engineering, Duke University

IV—4:00–5:00 P.M. Daily

(Coordination of) Fundamental Concepts in Mathematics (High School and College Level)
Leader: Professor L. S. Winton, Mathematics, North Carolina State College

V—9:30–10:30 A.M. Daily

Enrichment of Mathematics (High School and College Level)
Leader: Miss Veryl Schult, Supervisor Mathematics, Washington, D. C.

VI—4:00-5:00 P.M. Daily

Applications of Mathematics (High School and College Level, through Calculus)
Leader: Professor J. W. Cell, Mathematics, North Carolina State College

VII—9:30-10:30 A.M. Daily

The Slow Student in Mathematics (Junior and Senior High School Level)
Leader: Miss Mary A. Potter, Supervisor of Mathematics, Racine, Wis., Former President, National Council of Teachers of Mathematics

VIII—3:00-4:00 P.M. Daily

Making and Using Films in the Study of Mathematics (Junior and Senior High School Level)
Leader: Mr. W. Roger Zinn, Educational Consultant, Jam Handy Organization, Detroit, Mich.

The Second Annual Workshop for Teachers of Mathematics

June 28–July 2, 1948
Galesburg Division, University of Illinois,
Galesburg, Illinois

DR. DANIEL W. SNADER, *Director of the Workshop*

For application blanks or further information write: Robert B. Brown, Director of University Extension and Summer Session or Daniel W. Snader Galesburg Division
University of Illinois, Galesburg, Ill.
118 Illini Hall, Urbana, Ill.

Monday, June 28

MORNING SESSION—University Theater

9:00 A.M.

Official Greeting
Dean C. M. Louttit—Galesburg Division
Dean Robt. R. Browne—Director of Summer and Extension Divisions

Introduction of Workshop Staff
Prof. C. W. Sanford, Coordinator of Teacher Education

The Organization of Mathematics Study Groups*
Prof. Daniel W. Snader, Director of Workshop and Chairman of Division of Mathematics, Galesburg

Registration of Workshop Participants

11:30 A.M.

Luncheon—University Cafeteria

Address: "Education Looks Ahead."
Dean W. B. Spalding, Dean of the College of Education

AFTERNOON SESSION

1:30-3:30 P.M.

Afternoon Study Groups

Group II—Prof. John R. Clark, Prof. of Mathematical Education, Teachers College, Columbia University, New York, N. Y. (Room C-9-72)

Group IV—Miss Martha Hildebrandt, Head of Mathematics Department, Proviso Township High School, Maywood, Ill. (University Theater)

Group V—Seminar: Problems in the Teaching of High School Geometry
Topic: The Analytic Method in Teaching Geometry
Leader: Dr. Miles Hartley, Asst. Professor of Education, University of Ill. and Head Dept. of Mathematics, University High School (Room C-10-64)

3:30-6:00 P.M.—Recreation Period

Swimming, Billiards, Golf, Tennis, Pool, Bridge, etc.

6:30 P.M.—Dinner, University Cafeteria

EVENING SESSION—University Theater

7:45 P.M.

"The Mathematics Used in Watch Making"
Mr. B. L. Hummel, Head of Watch Design, Hamilton Watch Company, Lancaster, Pa.

Discussion

Tuesday, June 29

MORNING SESSION

10:00–12:00 A.M.

Morning Study Groups

Group I—Prof. John R. Clark. (Room C-9-72)
Group III—Prof. Daniel W. Snader. (Room C-10-64)

12:00 P.M.—Luncheon, University Cafeteria

AFTERNOON SESSION

1:30–3:30 P.M.

Afternoon Study Groups

Group II—Prof. John R. Clark. (Room C-9-72)

Group IV—Miss Martha Hildebrandt. (University Theater)

Group V—Seminar: Problems in the Teaching of High School Geometry
Topic: The Fundamentals of Critical Thinking in Geometric and Non-Geometric Situations
Leader: Miss Gertrude Hendrix, University High School, Eastern Illinois State Teachers College, Charleston, Ill. (Room C-10-64)

3:30–6:00 P.M.—Recreation Period

Swimming, Golf, Tennis, Billiards, Pool, Bridge, etc.

6:30 P.M.—Dinner, University Cafeteria

EVENING SESSION

7:30 P.M.

"Of What Value is the Monroe Calculator in Teaching Mathematics"
Mr. M. K. Patton, Representative Monroe Calculating Machine Co., Chicago, Ill.

9:15 P.M.—Social Evening, Faculty Lounge

Bridge Party and Refreshments

Wednesday, June 30

MORNING SESSION

10:00–12:00 A.M.

Morning Study Groups

Group I—Prof. John R. Clark. (Room C-9-72)
Group III—Prof. Daniel W. Snader. (Room C-10-64)

12:00 P.M.—Luncheon, University Cafeteria

AFTERNOON SESSION

1:30–3:30 P.M.

Afternoon Study Groups

Group II—Prof. John R. Clark. (Room C-9-72)

Group IV—Miss Martha Hildebrandt. (University Theater)

Group V—Seminar: Problems in the Teaching of High School Geometry
Topic: The Use of Instruments in Vitalizing the Teaching of Geometry
Leader: Mr. John F. Schacht, Head Dept. of Mathematics, Bixley High School, Columbus, Ohio (Room C-10-64)

3:30–6:00 P.M.—Recreation Period

Swimming, Golf, Tennis, Billiards, Pool, Bridge, etc.

6:30 P.M.—Banquet, University Cafeteria

Address: "Mathematics in the High School."
Prof. John R. Clark

EVENING SESSION

8:30 P.M.

"A Mathematical Analysis of Mechanisms Used in Automatic Machines"
John W. May, Chief Engineer of Design and Research, Wright Automatic Machinery Co., Durham, N. C.

Thursday, July 1

MORNING SESSION

10:00–12:00 A.M.

Group I—Prof. John R. Clark. (Room C-9-72)

Group III—Prof. Daniel W. Snader. (Room C-10-64)

12:00 P.M.—Luncheon, University Cafeteria

AFTERNOON SESSION

1:30–3:30 P.M.

Afternoon Study Groups

Group II—Prof. John R. Clark. (Room C-9-72)

Group IV—Miss Martha Hildebrandt. (University Theater)

Group V—Seminar: Problems in the Teaching of High School Geometry
Topic: The Testing and Evaluation Program for High School Geometry
Leader: Dr. Bjarne R. Ullsvik, Assoc. Professor of Mathematics, Illinois State Normal University, Normal, Ill. (Room C-10-64)

3:30–6:00 P.M.—Recreation Period (or Workshop Tea) (Faculty Lounge)
Swimming, Golf, Tennis, Billards, Pool, Bridge, etc.

6:30 P.M.—Dinner, University Cafeteria

EVENING SESSION

8:00 P.M.

"Some Application of Mathematics to the Problems of the Tire Industry"
Mr. R. D. Evans, Mathematician, Goodyear Tire and Rubber Company, Akron, Ohio

Friday, July 2

MORNING SESSION

10:00–12:00 A.M.

Morning Study Groups

Group I—Prof. John R. Clark. (Room C-9-72)

Group III—Prof. Daniel W. Snader. (Room C-10-64)

12:00 P.M.—Luncheon, University Cafeteria

AFTERNOON SESSION

1:30–3:00 P.M.

Afternoon Study Groups

Group II—Prof. John R. Clark. (Room C-9-72)
Group IV—Miss Martha Hildebrandt. (University Theater)
Group V—Seminar: Problems in the Teaching of High School Geometry
Topic: Curriculum Problems Related to High School Geometry
Leader: Dr. H. G. Ayre, Assoc. Professor of Mathematics, Western State Teachers College, Macomb, Ill.

3:15 P.M.—University Theater

Study Group Reports

Report of Committee on Recommendations

6:00 P.M.—Dinner, University Cafeteria

STUDY GROUPS

Morning Study Groups

Group I—*A Psychological Approach to the Teaching of Arithmetic*

An analysis of the types of learning in arithmetic, with suggestions for economical, effective achievement of each. One work period will be devoted to each of the following skills, meanings, problem solving, appreciations, and evaluation.

Group III—*Vitalized Instruction in Mathematics Through the Use of Instruments*

This group will study the use of the transit, level, sextant, plane table and alidade, angle mirror, hypsometer-clinometer, slide rule, computers, etc., in vitalizing the instruction in junior and senior high school mathematics. The mathematics of each instrument and the appropriate time to introduce each one in the regular sequence of mathematics courses, will be discussed. If you have a slide rule, bring it to the workshop.

Afternoon Study Groups

Group II—*A Psychological Approach to the Teaching of Algebra*

An analysis of the types of learning in algebra, with suggestions for economical effective achievement of each. One work period will be devoted to each of the following: skills, meanings, problem solving, appreciations, and evaluation.

Group IV—*Instructional Aids in the Teaching of Mathematics*

It is the aim of the group to study aids in teaching according to the following classifications: 1) those quickly made of easily available materials, 2) those aids which require more time and energy in planning and construction, 3) commercial aids and their use and finally 4) talking and moving pictures and recordings. Each member of the group should have a teaching unit in mind for which instructional aids are to be planned. Please bring along aids which you have found helpful in your work.

Group V—*Seminar: Problems in the Teaching of High School Geometry*

Monday: The Analytic Method in Teaching Geometry
Tuesday: The Fundamentals of Critical Thinking in Geometric and Non-Geometric Situations
Wednesday: The Use of Instruments in Vitalizing the Teaching of Geometry
Thursday: The Testing and Evaluation Program for High School Geometry
Friday: Curriculum Problems Related to High School Geometry

The National Council of Teachers of Mathematics Membership by States

	Jan. 1947	Jan. 1948		Jan. 1947	Jan. 1948
Alabama	79	77	Nebraska	79	78
Arizona	17	26	Nevada	4	10
Arkansas	64	58	New Hampshire	21	31
California	264	291	New Jersey	277	312
Colorado	80	75	New Mexico	18	42
Connecticut	119	114	New York	513	643
Delaware	23	24	North Carolina	103	96
D.C.	105	132	North Dakota	19	15
Florida	102	130	Ohio	348	331
Georgia	40	67	Oklahoma	97	81
Idaho	4	7	Oregon	77	80
Illinois	475	473	Pennsylvania	412	438
Indiana*	137	318	Rhode Island	37	36
Iowa	129	107	South Carolina	64	86
Kansas	166	150	South Dakota	21	25
Kentucky	69	68	Tennessee	88	97
Louisiana	80	100	Texas	200	208
Maine	34	28	Utah	13	10
Maryland	112	114	Vermont	16	27
Massachusetts	229	238	Viriginia	114	123
Michigan	241	213	Washington	76	67
Minnesota	138	144	West Virginia	43	52
Mississippi	34	42	Wisconsin	179	197
Missouri	133	122	Wyoming	19	26
Montana	20	24			

*** Indiana has more than doubled her membership in one year, while several other states are losing ground. Lets all renew our efforts.—Editor**

EDITORIAL

The Present Situation in Mathematics and What Should Be Done About It

THE secondary curriculum is now in pretty much the same state of affairs in which the world finds itself today—in a state of uncertainty, if not confusion. Many educators know that something is wrong, but most of them are afraid to go to the heart of the matter to seek out basic causes of the trouble, and screw up their courage to suggest what ought to be done to improve the situation. As a result, we have to stand by and listen to all kinds of panaceas for doing what needs to be done to remedy present defects, and to reorganize the curriculum so as more nearly to meet the needs and interests of large numbers of pupils in the secondary school.

The responsibility for the situation described above rests not only on those who administer the secondary schools, but also upon the colleges who are ultimately to receive many of the graduates of the secondary schools. Many college authorities have shown little interest in improving the curriculum in schools which furnish them with most of their students. They have permitted unqualified and undesirable students to enroll in the colleges without giving much serious thought to the quality of their high school work. Often when they discover with some surprise that students do not have proper collegiate background, they proceed to give them glorified courses in secondary school subjects. This is particularly true in mathematics where, in the absence of proper guidance, students are permitted to graduate from high school with a minimum of mathematical training.

If it can be arranged, there ought to be formed a joint commission composed of the best minds in the National Council of Teachers of Mathematics and the Mathematical Association of America, who are really interested in the future of mathematics in this country, and who are not so "steeped in tradition that they cannot approach the problem with an open mind."

What we need just now is a leveling up instead of a leveling down. Boys and girls who are clearly of college calibre should be given the type of mathematics training that will best fit them for college and the kind of life work they plan to do. Moreover, the boys and girls who are not of college calibre should be given the kind of course which will best equip them for their life work. But we must be careful in choosing the students for each track concerned.

It is clear now that the extra-mural examining bodies in this country, the College Entrance Examination Board, the New York Regents, and similar systems, if any exist, are not solving the problem. We have improved syllabi from year to year, from one period to another, but the bad effects of the examination system still persist. The results in the Regents system recently in history were so bad that the passing mark had to be lowered before a significant number of pupils could be given Regents' credit. Why the teachers of the various subjects are willing to continue paying lip service to the Regents examinations is hard to understand, when the entire situation is in such need of reform.

Dr. Betz' article in this issue of THE MATHEMATICS TEACHER gives many facts of the present unsatisfactory situation in the schools. It should be read by all those who are interested in a better program of education for American youth.—W. D. R.

IN OTHER PERIODICALS

By NATHAN LAZAR

Midwood High School, Brooklyn 10, New York

The American Mathematical Monthly

March 1948, Vol. 55, No. 3

1. MacDuffee, C. C., "The Scholar in a Scientific World," pp. 129–140.
2. Isaacs, Rufus, "Recent Progress in Compressible Fluid Theory," pp. 140–144.
3. Jones, P. S., "The Teaching Fellow Program at Michigan," pp. 145–147.
4. Davis, Chandler, "The Short-Cut Problem," pp. 147–150.
5. Mathematics Notes, pp. 152–153.
   Butchart, J. H., "Rotation of the Tangent to a Hypochloid."
6. Classroom Notes, pp. 153–158.
   a. Duffin, R. J., "A Generalization of the Ratio Test for Series."
   b. May, Kenneth, "Derivation of the Normal Form of the Equation of the Straight Line."
   c. Young, F. H., "The Ellipse as a Circle with a Moving Center."

School Science and Mathematics

January 1948, Vol. 48, No. 1

1. Reeve, W. D., "Modern Trends in Mathematics Education," pp. 21–33.
2. Edward, R. L., "The Unappreciated Usefulness of Mathematical Composition," pp. 51–54.
3. Nyberg, Joseph A., "Notes from a Mathematics Classroom" (continued) pp. 54–57.

February 1948, Vol. 48, No. 2

1. Bacon, Ralph Hoyte, "Integral Solutions of $x^2+y^2+z^2=r^2$," pp. 155–164.
2. Nyberg, Joseph A., "Notes from a Mathematics Classroom" (continued) pp. 191–194.

Miscellaneous

1. Adams, O., "Arithmetic Readiness in the Primary Grades," *Elementary School Journal*, 48: 91–96, October 1947.
2. Brownell, W. A., "Experiment on Borrowing in Third-Grade Arithmetic," *Journal of Educational Research*, 41: 161–171, November 1947.
3. Brueckner, L. J., "Experience Units in Arithmetic," *National Education Association Journal*, 37: 20–21, January 1948.
4. Buswell, G. T., "Selected References on Elementary-School Instruction: Arithmetic." *Elementary School Journal*, 48: 155–157, November 1947.
5. Buswell, G. T., "Scholarship in Elementary-School Teaching," *Elementary School Journal*, 48: 242–244, January 1948.
6. Cassell, M., "House that Arithmetic Built," *The Grade Teacher*, 65: 20–21 +, September 1947.
7. Cooke, R. J., "Develop Arithmetic Readiness," *Instructor*, 57: 17 +, January 1948.
8. Davis, R. A., and Rood, E. J., "*Remembering and Forgetting Arithmetical Abilities,*" *Journal of Educational Psychology*, 38: 216–222, April 1947.
9. Delva, J. D., "Planning our Homes; Activity in Social Arithmetic," *The Grade Teacher*, 65: 10 +, October 1947.
10. Earley, A., "Checking the Four Fundamental Operations," *The Grade Teacher*, 64: 50, May 1947.
11. Foran, T. G., "Should Reading or Arithmetic Be Introduced First in the Primary Grades?" *Proceedings, National Catholic Educational Association*, 1947: 458–461.
12. Hall, J. V., "Solving Verbal Arithmetic Problems Without Pencil and Paper," *Elementary School Journal*, 48: 212–217, December 1947.
13. Halliday, F. F., "Arithmetic, Cut From the Child," *School* (Elementary Edition), 36: 121–123, November 1947.
14. Morton, R. L., "Securing Better Results in Arithmetic," *National Educational Association Journal*, 36: 568–569, November 1947.
15. Morton, R. L., "Estimating Quotient Figures When Dividing by Two-Place Numbers," *Elementary School Journal*, 48: 141–148, November 1947.
16. Stephens, J. M., and Lichtenstein, A., "Factors Associated with Success in Teaching Grade Five Arithmetic," *Journal of Educational Research*, 40: 683–694, May 1947.
17. Taylor, E. R., "Adapting the Early Work in Arithmetic to the Child's Interest," *American Childhood*, 33: 20–21, September 1947.
18. Wurman, C. G., and Nadler, M., "Arithmetic Through Application," *High Points*, 29: 68–71, November 1947.

National Council Yearbook!

The National Council Yearbooks are rapidly going out of print. Teachers who wish complete files and, particularly, school libraries who lack certain books should order now.—EDITOR.

NEWS NOTES

1948 Summer Session

Teachers College, Columbia University

Courses in the Teaching of Mathematics

Teachers College, Columbia University, will offer the following courses in the teaching of mathematics in the summer session of 1948, which begins on July 6 and ends on August 13:

By Professor John R. Clark: Teaching arithmetic in the elementary school.

By Professor Howard F. Fehr: Professionalized subject matter in advanced secondary school mathematics; teaching algebra in secondary schools.

By Dr. Nathan Lazar: History of mathematics; logic for teachers of mathematics.

By Mr. Gordon R. Mirick: Elementary mechanics (kinetics); teaching geometry in secondary schools.

By Professor William D. Reeve: Teaching and supervision of mathematics—junior high school; teaching and supervision of mathematics—senior high school.

By Professor Carl N. Schuster: Modern business arithmetic; field work in mathematics.

There will be held during the summer session, on consecutive Thursdays beginning on July 8, five special lectures and discussions in which all of the instructors above and other persons from outside will participate for the purpose of bringing before the students vital questions relating to the reorganization and teaching of mathematics in the post-war world. There will be an opportunity for discussion in which all of the students will be invited to take part. These conferences have come to be a common meeting place of all students, instructors, and guests, and thus serve both professional and social ends. Registration for these special lectures and discussions is not necessary and all those who are interested in the improvement of mathematical education are invited to attend.

The Suffolk County, N. Y., Mathematics Teachers Association has held two meetings so far this year under the presidency of Mr. Charles Kissam. The first was held in November at Northport with Dr. Barravalle speaking on "The Dynamic Beauty to Geometry." The second meeting was held in December in Sayville where Miss Edina Van Wart, Mrs. Harriet Burgie, Miss Alice Griswold, and Miss Leona Hirzel reported on "The Duke Institute for Mathematics Teachers."

Those members present were from the following schools: Babylon, Bayport, Huntington, Northport, Patchogue, Sayville and Southampton.

University of Virginia

Conference of Mathematics Teachers

February 20 and 21, 1948

Friday, February 20

PROGRAM

Morning Session: G. T. Whyburn, Professor of Mathematics, University of Va., presiding

10:15–10:30. Welcome: Dean Ivey F. Lewis, University of Virginia.

10:30–11:30. Let's Face the Guidance Problem in Mathematics Education—Raleigh Schorling, Head Dept. of Mathematics, University High School, and Professor of Education, University of Michigan.

11:00–11:30. The Nature of Mathematics and Mathematical Proofs—E. J. McShane, Professor of Mathematics, University of Virginia.

11:30–12:30. Round Table Discussions.

Group I. Mathematics in the New Eighth Grade
Leader: Geo W. Cox, Altairsta High School.

Group II. Selecting Students for the Elective Courses
Leader: Lucy Sinclair, Newport News High School

Group III. What Shall the Total Offering Be?
Leader: Raleigh Schorling

Group IV. What Tests Should We Give?
Leader: A. L. Wingo, State Dept. of Education.

12:30–2:00. Lunch

Afternoon Session: Dean J. L. Manahan, University of Virginia, presiding

2:00–2:30. What is Good Preparation in Mathematics for College?—G. A. Hedlund, Professor of Mathematics, University of Va.

2:30–3:00. Meeting the Mathematics Needs of Pupils Who Will Not Go to College.—F. G. Lankford, Jr., Assoc. Prof. of Education, University of Virginia.

3:00–4:00. Round Table Discussions.

Group I.; Teaching Verbal Problems in Algebra
Leader: To be selected.

Group II. Stimulating Interest in Mathematics Through Field Work.
Leader: W. S. Rumbough, Falls Church High School.

Group III. Supplementary Assignments for the Bright Pupil.
Leader: H. H. Walker, Lone High School.

Group IV. Minimum Essentials in General Mathematics
Leader: Raleigh Schorling.

4:30–5:30. Social hour.

Saturday, February 21

F. G. Lankford, Jr., presiding

9:00–9:45. What's Going on in Your School?—The Mathematics Inquiry. Raleigh Schorling.

9:45–10:30. Projected Visual Aids in Mathematics Teaching—J. A. Rorer, Assoc. Prof. of Education, University of Virginia.

10:30–11:15. Non-Projected Visual Aids in Mathematics Teaching. Speaker: Allene Archer, Thos. Jefferson High School, Richmond.

11:15–12:30. Demonstrations of Visual Aids in Mathematics.

Group I. Projected Aids—Dr. Alex Rarer.

Group II. Non-Projected Aids. Allene Archer.

Texas Section

Mathematical Association of America

Worth Hotel, Fort Worth, Texas

PROGRAM

2–5 P.M. Friday, December 6, 1946

1. "A Plane Version of Solid Analytical Geometry," R. S. Underwood, Texas Tech. College, 15 min.
2. "Note on the Zeros of $P_n^m(\cos\theta=0$ and $dP_n^m(\cos\theta)/d\theta=0$ considered as Functions of n," C. W. Horton, Defense Research Laboratory Staff, University of Texas, 15 min.
3. "Recent High Speed Methods of Computation Developed for War Research," H. J. Ettlinger, University of Texas.
4. Paper (by request) R. E. Langer, University of Texas, 1 hour.

7 P.M. Friday, December 6, 1947

Dinner (informal), Worth Hotel

Address: "The Role of Mathematics in the Post World War II Educational Programs," W. M. Whyburn, President, Texas Tech. College

9–12 A.M. Saturday, December 7, 1947

Round Table Discussion of Teaching Problems in Mathematics

High Schools: Miss Elizabeth Dice, Dallas, Texas

Junior Colleges: Miss Mabel Williams, Tyler Junior Collepe

Senior Colleges: W. L. Porter, Texas A & M

The Women's Mathematics Club of Chicago and vicinity met at Mandel's Tea Room on February 7, 1948.

The speaker was Professor Mason E. Wescott of Northwestern University who spoke on the topic "The Construction and Interpretation of Average and Range Charts in Statistical Quality Control."

The Fourth Meeting of the Men's Mathematics Club of Chicago and the metropolitan area was held at the Central Y.M.C.A. on Jan. 16, 1948.

Dr. Karl Menger, Professor of Mathematics at the Illinois Institute of Technology spoke on the topic "What is a Curve?"

BOOK REVIEWS

Theory of Functions. By Joseph Fels Ritt. King's Crown Press, N.Y. 1947. Revised edition, x+181 pp, $8\frac{1}{2}\times 11$. Loose leaf, paper bound, lithoprinted. $3.00.

This book contains essentially the material given by the author in his courses at Columbia University. It includes both the real and complex variable, about sixty per cent of the book dealing with the complex function. In developing the real number system, Professor Ritt departs from the traditional approach of Cantor's infinite sequences of Dedekind's cut, using infinite decimals in place. This method is quite simple and has the advantage of familiarity to the student.

The usual topics treated in systematic order include:—the theory of limits, linear point sets, continuity, the derivative, Riemann integration, infinite sequences and series of numbers and of functions, functions of two variables, complex numbers, point sets in a complex plane, Curves, curvilinear integrals, Jordan curves, analysis situs of the triangle, the Cauchy integral theorem, Cauchy's integral formula, analytic functions, Taylor's series, Liouville's theorem, Laurent's series, rational functions, the Mittag-Leffler theorem, residues, and analytic continuation.

The simplicity of exposition, and the choice of pertinent examples and illustrations are outstanding. The statement of pertinent examples and illustrations are outstanding. The statement of the theorems and the phraseology in the proof of theorems possess a clarity that is all too rare in most advanced treatises. If one desires to give a course in function theory, not specifically his own creation, or if one desires to review his theory, this volume offers an excellent opportunity for real study.—HOWARD F. FEHR.

Whom the Gods Love: The Story of Evariste Galois. By Leopold Infeld. Whittlesey House, N. Y. ix+323 pages. Price $3.50.

Leopold Infeld, a physicist and scientific writer, is perhaps best known for *The Evolution of Physics* of which he is the co-author with Albert Einstein. In *Whom the Gods Love* he has turned his hand to a different style and produced an absorbing fictionalized biography. Little is known of the details of Galois' life and thus the author has availed himself of his literary privilege to fill in the gaps between the established facts. The result is as much the story of the turbulent period in French history following the Revolution as it is of the short and tragic life of the genius of mathematical group-theory.

Apart from its literary value, the book is of special interest to mathematicians and educators. Although Mr. Infeld does not discuss the mathematical aspects of Galois' work, except descriptively, his account of the young mathematician's school experiences is most illuminating. Many of the evils of modern education are seen, in exaggerated form, in the French schools which Galois attended and also in those to which he was not admitted because of shortsighted entrance requirements.

Finally, the social and political significance of the book must not be overlooked, for it has particular relevance to the ideological conflicts which are ravaging the world today.—FREDERIC W. BORGES

Integration in Finite Terms. By Joseph Fels Ritt. Columbia University Press, New York 1948. ix+100 pages. $2.75.

This volume is the result of work done by the author in a series of studies from 1923 to 1927. The main purpose of the treatise is to give an exposition of Liouville's problem on determining the form of the integrand of an algebraic function so that the integral can be expressed with a finite number of operations of elementary analysis. The treatment assumes on the part of the reader a thorough knowledge of the calculus, differential equations, and theory of functions of a complex variable, especially analytic functions and analytic continuation.

The first chapter deals with an investigation into all types of functions and the differentiation of these functions. In turn, the author then treats:—integrals of algebraic functions, integrals of transcendental functions, series of fractional powers, integration of differential equations by quadratures, and implicit and explicit solutions of differential equations of the first order. The book should be of particular interest to the specialist in analysis and in function theory.—HOWARD F. FEHR.

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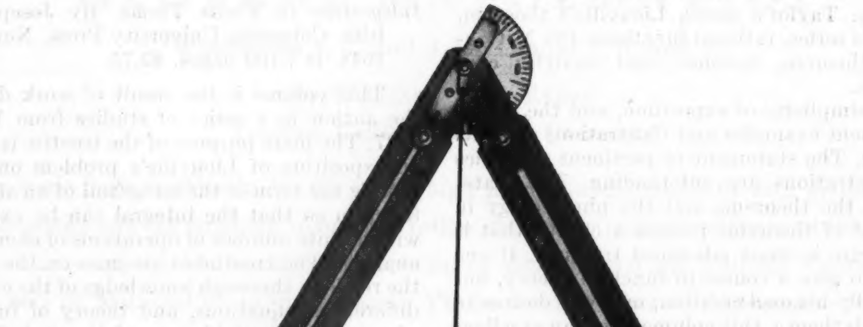

Figure 1

not needed. One method of showing the division of 132 by 3 might be this:

$$3\overline{)132}$$

"I cannot divide 1 by 3. So I examine 13. Since $3\times 4=12$, $13\div 3$ is 4."

"Since I am dividing 13 by 3, the quotient 4 is written above the 3 of 13." The child then multiplies, subtracts, compares, brings down, and completes his division, placing the last figure of the quotient above the last figure of his divisor. The answer is 44. The child checks by multiplying 44 by 3. Knowledge of place value has not been used to confuse him. The method is straight forward and simple.

Let us consider another method of presenting this problem, a method proposed by some educators. "We are to divide 1 hundred, 3 tens, and 2 units by 3 units. But 1 hundred cannot be divided by 3 units. (Of course we know that 100 can be divided by 3. The statement of truth is more subtle than that. It is: 'The *number* of hundreds cannot be divided by the *number* of units.' I doubt that many children will see the distinction.) "So we examine 1 hundred and 3 tens, and call this number 13 tens." (Another skill is called for here which in no way helps in the development of the algorism nor in giving the child understanding of division.) "Then 13 tens divided by 3 units is 4 tens." (Will the child know that the result is tens, and not units?)

"Multiply 4 tens by 3 tens and get 12 tens." (Will he know it is tens and not

$$\begin{array}{r} 31 \\ 7\overline{)28} \\ 7 \\ \hline 21 \\ 21 \\ \hline \end{array}$$

"28 is 2 tens and 8 units. I divide the units first. 8 units divided by 7 units is 1 unit which I place in the units' column above 8. 1 unit times 7 units is 7 units which I subtract from 8 units and get 1 unit. Bring down the 2. Then $21\div 7$ is 3, which I place in the ten's column since 21 is 2 tens and 1 unit."

The confusion in this development is even more evident when there is a two figure divisor as $23\overline{)115}$. Again we have the subtle statement: "1 hundred cannot be divided by 2 tens. Then call 1 hundred and 1 ten, 11 tens. 11 tens$\div$2 tens is 5 units." The use of "tens, hundreds, etc., as concrete numbers instead of as abstract numbers, necessary by this method as in dividing $200\overline{)6800}$ where the child must answer the question: 6 ten thousands divided by 2 hundreds is what? (Can you answer this, off the bat?) shows that there is little of real meaning and understanding to be transferred for later use by the child. And division by 3 figure numbers is even worse. But it is required in the junior high school in problems in mensuration, making graphs, and in the use of money.

Certainly place value must be taught, but always within the child's ability to understand. He must not be asked to make generalizations and abstractions that would puzzle many adults. And the learning, the meaning, must be easily recalled in later years.

4. *Every teacher of arithmetic should know well how arithmetic is taught in the lower grades.*

current use of arithmetic, as he gets farther away from the eighth grade. In a similar study at Montclair, the same results were obtained with pupils in the college preparatory course, showing that college preparatory mathematics does not provide the practice in number needed to maintain arithmetic skills.

If a pupil in any grade has forgotten facts or processes in mathematics, the only recourse is for the teacher to explain them, giving them meaning, and reteach them at that time. No valuable end is served if the teacher wrings his hands and tries to place the blame on earlier teachers, on the parents, or on Providence. This reteaching, this putting meaning into forgotten processes, can only be done by a teacher who has studied how arithmetic can be taught with understanding.

5. *At all stages in the teaching of arithmetic the development must begin with the concrete then go to the abstract and finally be applied in the concrete situations of real life.*

Plato gave good advice when he said that to teach children how to count, begin with objects familiar to them: apples, their toys, balls. So we begin with concrete objects. The child counts pencils, chairs, children, and beads on the abacus. When he learns to say the numbers "one, two, three, four, five" by rote, he does not necessarily know how to count. He must be able to place these number names in one-to-one correspondence with the five apples he is counting. Until he has learned to abstract so far as to place his finger on one apple and say "One," on a different

degree of abstraction in so simple a process as counting, it becomes plain why teachers must understand the difficulties involved and must realize that there can be no mastery of arithmetic unless the child understands.

We begin with concrete objects and situations and continue with them until the child can make the abstractions required. The task is not complete until the child is able to apply what he has learned in concrete every day life situations. He must begin with the concrete and end with it. In between he must abstract.

6. *Understanding is hindered by the use of words or devices that conceal mathematical meaning.*

Let us take the word "cancel" as an example. There is no excuse for introducing the word "cancel," a word which may mean either subtraction as in 6–6 where the 6's cancel each other and give zero or may mean division as in 6/6 where the 6's cancel and give 1. There are enough abstractions in arithmetic without the use of confusing terms like cancellation. $6-6$ is equal to zero. There is no cancelling. There *is subtraction.* 6/6 is equal to one. The numerator and denominator are each divided by 6.

Bright Barbara brought her homework for dad to examine. She had changed 250/450 to per cent and had 50% for an answer. She explained how she did it. "First I cancel the zeros and get 25/45. Then I cancel the 5's and get 2/4 or 1/2. That is 50%."

If one is going to cancel, that is bright work, showing that the child could ab-